Keystone Exams: Geometry

Assessment Anchors and Eligible Content

with Sample Questions and Glossary

Pennsylvania Department of Education

www.education.state.pa.us

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PENNSYLVANIA DEPARTMENT OF EDUCATION

General Introduction to the Keystone Exam Assessment Anchors

Introduction

Since the introduction of the Keystone Exams, the Pennsylvania Department of Education (PDE) has been working to create a set of tools designed to help educators improve instructional practices and better understand the Keystone Exams. The Assessment Anchors, as defined by the Eligible Content, are one of the many tools the Department believes will better align curriculum, instruction, and assessment practices throughout the Commonwealth. Without this alignment, it will not be possible to significantly improve student achievement across the Commonwealth.

How were Keystone Exam Assessment Anchors developed?

Prior to the development of the Assessment Anchors, multiple groups of PA educators convened to create a set of standards for each of the Keystone Exams. Enhanced Standards, derived from a review of existing standards, focused on what students need to know and be able to do in order to be college and career ready. (Note: Since that time, PA Core Standards have replaced the Enhanced Standards and reflect the college- and career-ready focus.) Additionally, the Assessment Anchors and Eligible Content statements were created by other groups of educators charged with the task of clarifying the standards assessed on the Keystone Exams. The Assessment Anchors, as defined by the Eligible Content, have been designed to hold together, or *anchor*, the state assessment system and the curriculum/instructional practices in schools.

Assessment Anchors, as defined by the Eligible Content, were created with the following design parameters:

- **Clear:** The Assessment Anchors are easy to read and are user friendly; they clearly detail which standards are assessed on the Keystone Exams.
- **Focused:** The Assessment Anchors identify a core set of standards that can be reasonably assessed on a large-scale assessment; this will keep educators from having to guess which standards are critical.
- **Rigorous:** The Assessment Anchors support the rigor of the state standards by assessing higher-order and reasoning skills.
- **Manageable:** The Assessment Anchors define the standards in a way that can be easily incorporated into a course to prepare students for success.

How can teachers, administrators, schools, and districts use these Assessment Anchors?

The Assessment Anchors, as defined by the Eligible Content, can help focus teaching and learning because they are clear, manageable, and closely aligned with the Keystone Exams. Teachers and administrators will be better informed about which standards will be assessed. The Assessment Anchors and Eligible Content should be used along with the Standards and the Curriculum Framework of the Standards Aligned System (SAS) to build curriculum, design lessons, and support student achievement.

The Assessment Anchors and Eligible Content are designed to enable educators to determine when they feel students are prepared to be successful in the Keystone Exams. An evaluation of current course offerings, through the lens of what is assessed on those particular Keystone Exams, may provide an opportunity for an alignment to ensure student preparedness.

How are the Assessment Anchors organized?

The Assessment Anchors, as defined by the Eligible Content, are organized into cohesive blueprints, each structured with a common labeling system that can be read like an outline. This framework is organized first by module, then by Assessment Anchor, followed by Anchor Descriptor, and then finally, at the greatest level of detail, by an Eligible Content statement. The common format of this outline is followed across the Keystone Exams.

Here is a description of each level in the labeling system for the Keystone Exams:

- **Module:** The Assessment Anchors are organized into two thematic modules for each of the Keystone Exams. The module title appears at the top of each page. The module level is important because the Keystone Exams are built using a module format, with each of the Keystone Exams divided into two equal-size test modules. Each module is made up of two or more Assessment Anchors.
- **Assessment Anchor:** The Assessment Anchor appears in the shaded bar across the top of each Assessment Anchor table. The Assessment Anchors represent categories of subject matter that anchor the content of the Keystone Exams. Each Assessment Anchor is part of a module and has one or more Anchor Descriptors unified under it.
- **Anchor Descriptor:** Below each Assessment Anchor is a specific Anchor Descriptor. The Anchor Descriptor level provides further details that delineate the scope of content covered by the Assessment Anchor. Each Anchor Descriptor is part of an Assessment Anchor and has one or more Eligible Content statements unified under it.
- **Eligible Content:** The column to the right of the Anchor Descriptor contains the Eligible Content statements. The Eligible Content is the most specific description of the content that is assessed on the Keystone Exams. This level is considered the assessment limit and helps educators identify the range of the content covered on the Keystone Exams.
- **PA Core Standard:** In the column to the right of each Eligible Content statement is a code representing one or more PA Core Standards that correlate to the Eligible Content statement. Some Eligible Content statements include annotations that indicate certain clarifications about the scope of an Eligible Content.

    - "e.g." ("for example")—sample approach, but not a limit to the Eligible Content
    - "Note"—content exclusions or definable range of the Eligible Content

How do the K–12 Pennsylvania Core Standards affect this document?

Assessment Anchor and Eligible Content statements are aligned to the PA Core Standards; thus, the former enhanced standards are no longer necessary. Within this document, all standard references reflect the PA Core Standards.

Standards Aligned System—www.pdesas.org

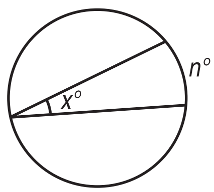
Pennsylvania Department of Education—www.education.state.pa.us

Formula Sheet

Formulas that you may need to work questions in this document are found below.
You may use calculator π or the number 3.14.

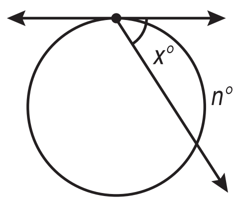
Properties of Circles

Angle measure is represented by *x*. Arc measure is represented by *m* and *n*. Lengths are given by *a*, *b*, *c*, and *d*.



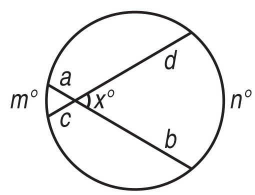
Inscribed Angle

$$x = \frac{1}{2}n$$



Tangent-Chord

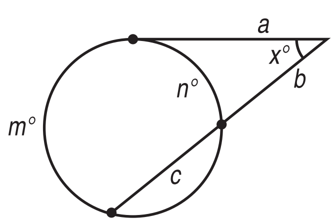
$$x = \frac{1}{2}n$$



2 Chords

$$a \cdot b = c \cdot d$$

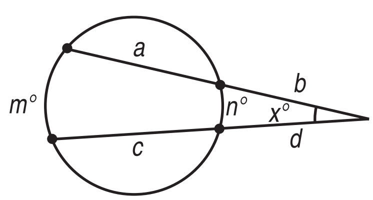
$$x = \frac{1}{2}(m + n)$$



Tangent-Secant

$$a^2 = b(b + c)$$

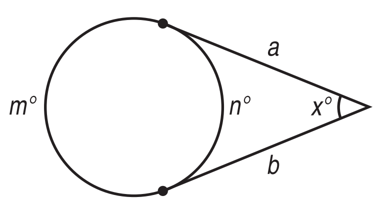
$$x = \frac{1}{2}(m - n)$$



2 Secants

$$b(a + b) = d(c + d)$$

$$x = \frac{1}{2}(m - n)$$

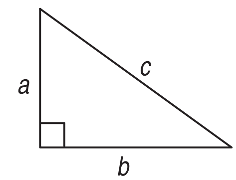


2 Tangents

$$a = b$$

$$x = \frac{1}{2}(m - n)$$

Right Triangle Formulas

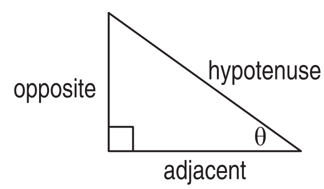


Pythagorean Theorem:

If a right triangle has legs with measures *a* and *b* and hypotenuse with measure *c*, then...

$$a^2 + b^2 = c^2$$

Trigonometric Ratios:



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Coordinate Geometry Properties

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-Slope Formula: $(y - y_1) = m(x - x_1)$

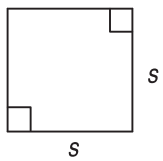
Slope Intercept Formula: $y = mx + b$

Standard Equation of a Line: $Ax + By = C$

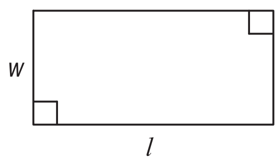
Formula Sheet

Formulas that you may need to work questions in this document are found below.
You may use calculator π or the number 3.14.

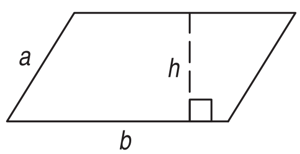
Plane Figure Formulas



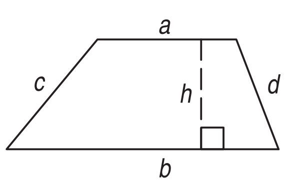
$P = 4s$
$A = s \cdot s$



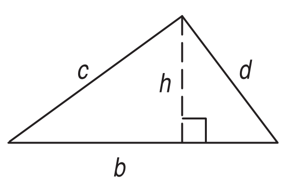
$P = 2l + 2w$
$A = lw$



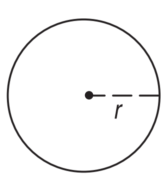
$P = 2a + 2b$
$A = bh$



$P = a + b + c + d$
$A = \frac{1}{2}h(a + b)$



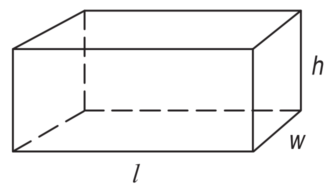
$P = b + c + d$
$A = \frac{1}{2}bh$



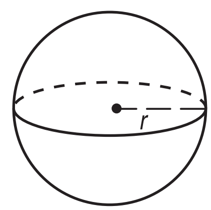
$C = 2\pi r$
$A = \pi r^2$

Sum of angle measures = $180(n - 2)$,
where n = number of sides

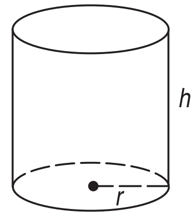
Solid Figure Formulas



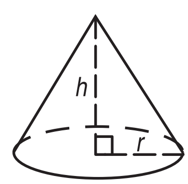
$SA = 2lw + 2lh + 2wh$
$V = lwh$



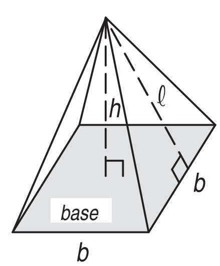
$SA = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$



$SA = 2\pi r^2 + 2\pi rh$
$V = \pi r^2 h$



$SA = \pi r^2 + \pi r\sqrt{r^2 + h^2}$
$V = \frac{1}{3}\pi r^2 h$



SA = (Area of the base) + $\frac{1}{2}$(number of sides)(b)(ℓ)

$V = \frac{1}{3}$(Area of the base)(h)

Euler's Formula for Polyhedra:

$V - E + F = 2$

vertices minus edges plus faces = 2

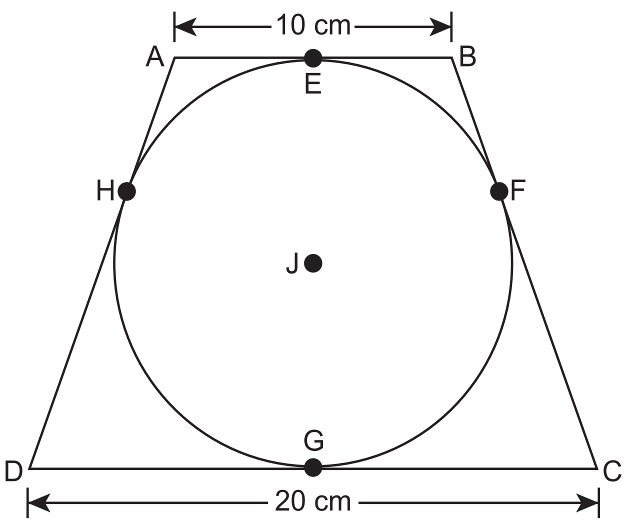
MODULE 1—Geometric Properties and Reasoning

ASSESSMENT ANCHOR G.1.1 Properties of Circles, Spheres, and Cylinders				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.1.1.1	Identify and/or use parts of circles and segments associated with circles, spheres, and cylinders.	**G.1.1.1.1**	Identify, determine, and/or use the radius, diameter, segment, and/or tangent of a circle.	**CC.2.3.HS.A.8** **CC.2.3.HS.A.9** **CC.2.3.HS.A.13**
		G.1.1.1.2	Identify, determine, and/or use the arcs, semicircles, sectors, and/or angles of a circle.	
		G.1.1.1.3	Use chords, tangents, and secants to find missing arc measures or missing segment measures.	
		G.1.1.1.4	Identify and/or use the properties of a sphere or cylinder.	

Sample Exam Questions

Standard G.1.1.1.1

Circle J is inscribed in isosceles trapezoid ABCD, as shown below.



Points E, F, G, and H are points of tangency. The length of $\overline{AB}$ is 10 cm. The length of $\overline{DC}$ is 20 cm. What is the length, in cm, of $\overline{BC}$?

A. 5

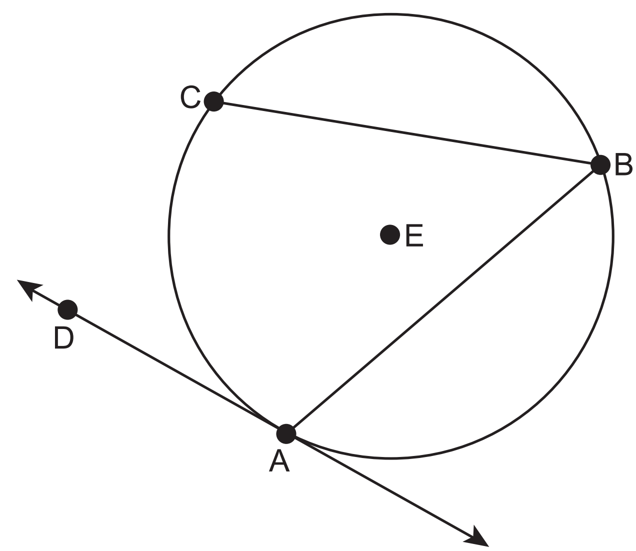
B. 10

C. 15

D. 30

Standard G.1.1.1.2

Circle E is shown in the diagram below.



Line AD is tangent to circle E. The measure of angle DAB is 110°. The measure of minor arc CB is 120°. What is the measure of arc CBA?

A. 220°

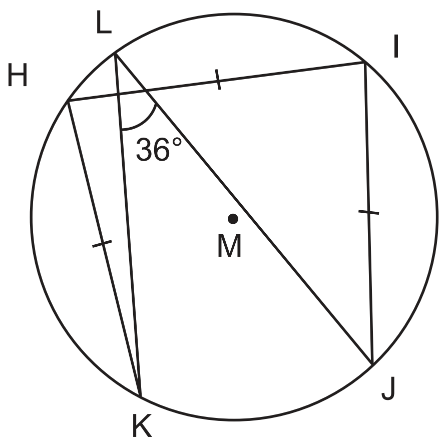
B. 240°

C. 250°

D. 260°

Standard G.1.1.1.3

Circle M is shown below.



Chords $\overline{KH}$, $\overline{HI}$, and $\overline{IJ}$ are congruent. What is the measure of $\overset{\frown}{KH}$?

A. 72°

B. 90°

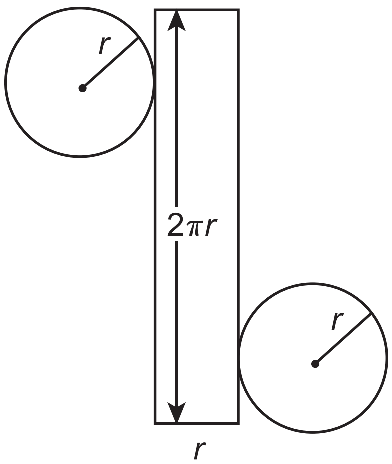
C. 96°

D. 108°

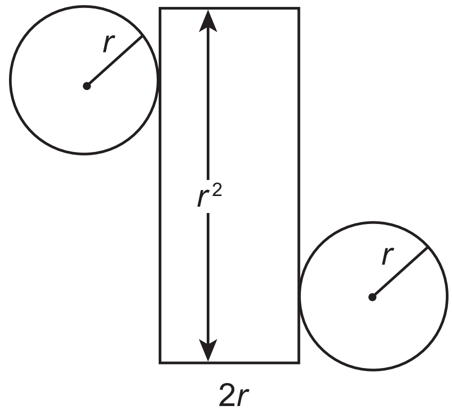
Standard G.1.1.1.4

Which of the nets shown below represents a cylinder for all positive values of r?

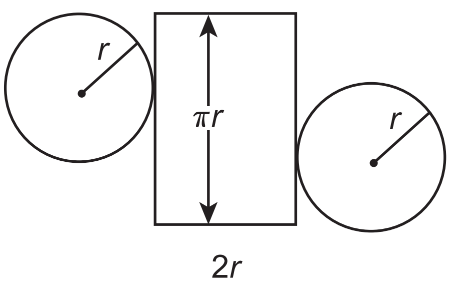
A.



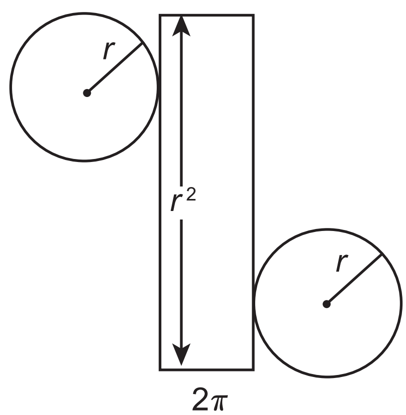
B.



C.



D.

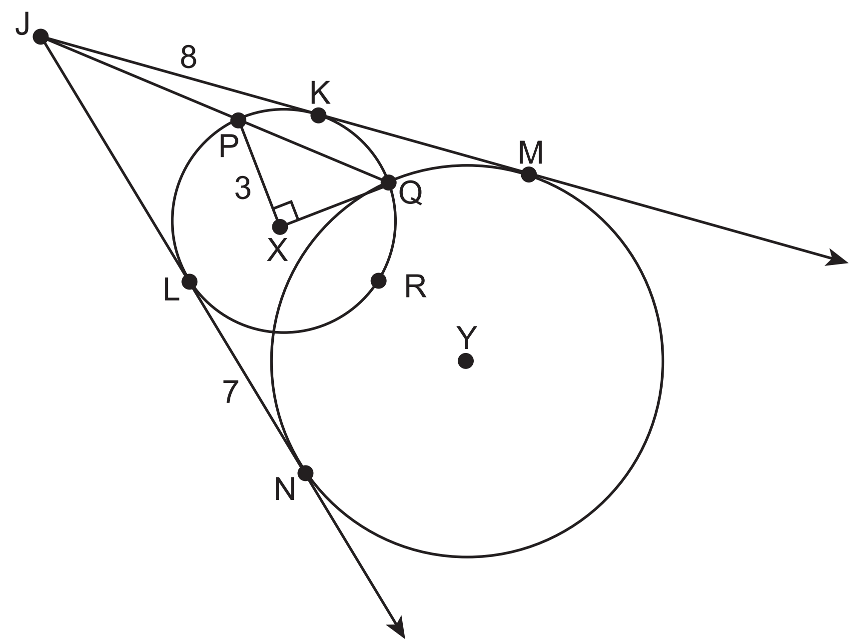


ASSESSMENT ANCHOR	
G.1.1	Properties of Circles, Spheres, and Cylinders

Sample Exam Questions

Standard G.1.1

A diagram is shown below.



In the diagram, $\overline{JM}$ and $\overline{JN}$ are tangent to circle X and circle Y.

A. What is the length of $\overline{JM}$?

length of $\overline{JM}$: ______________________

Continued on next page.

Continued. Please refer to the previous page for task explanation.

B. Identify the chord in the diagram.

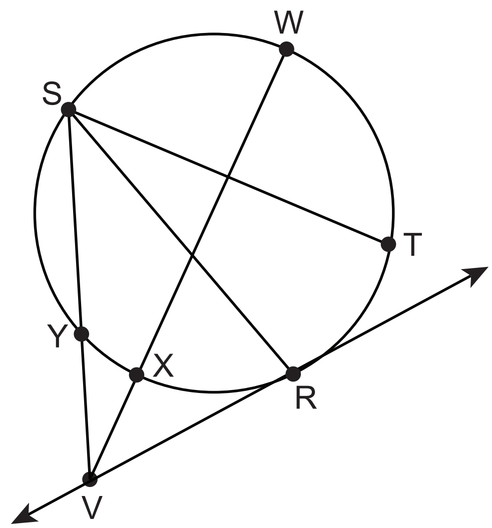
chord: ____________________

C. What is the length of $\overline{JP}$? Show your work. Explain your reasoning.

MODULE 1—Geometric Properties and Reasoning

Standard G.1.1

A circle is shown below.



Some information about the circle is listed below.

- $\overleftrightarrow{VR}$ is tangent to the circle
- $m\angle VRS = 77°$
- $m\angle RST = 27°$
- $m\overset{\frown}{SW} = 78°$
- $m\overset{\frown}{XY} = 22°$
- $m\overset{\frown}{SY} = 78°$

A. What is the measure of $\angle XVY$?

$m\angle XVY =$ ______________________________

Continued on next page.

Continued. Please refer to the previous page for task explanation.

B. What is the measure of $\overparen{RT}$?

$m\ \overparen{RT}$ = ____________________________

C. What is the measure of $\overparen{WT}$?

$m\ \overparen{WT}$ = ____________________________

D. What is the measure of $\angle SVR$?

$m\ \angle SVR$ = ____________________________

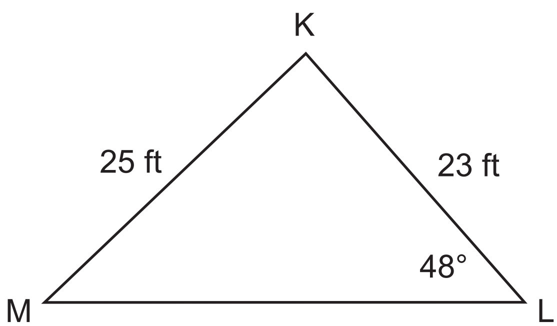
MODULE 1—Geometric Properties and Reasoning

ASSESSMENT ANCHOR **G.1.2** Properties of Polygons and Polyhedra				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.1.2.1	Recognize and/or apply properties of angles, polygons, and polyhedra.	**G.1.2.1.1**	Identify and/or use properties of triangles.	**CC.2.3.8.A.2** **CC.2.3.HS.A.3** **CC.2.3.HS.A.13**
		G.1.2.1.2	Identify and/or use properties of quadrilaterals.	
		G.1.2.1.3	Identify and/or use properties of isosceles and equilateral triangles.	
		G.1.2.1.4	Identify and/or use properties of regular polygons.	
		G.1.2.1.5	Identify and/or use properties of pyramids and prisms.	

Sample Exam Questions

Standard G.1.2.1.1

Acute triangle KLM is shown below.



Which could be the measure of $\angle M$?

A. 38°

B. 42°

C. 44°

D. 52°

Standard G.1.2.1.2

Quadrilateral WXYZ is a kite. Which of the following **must** be true?

A. $\overline{WX}$ and $\overline{YZ}$ are congruent

B. $\overline{WY}$ and $\overline{XZ}$ bisect each other

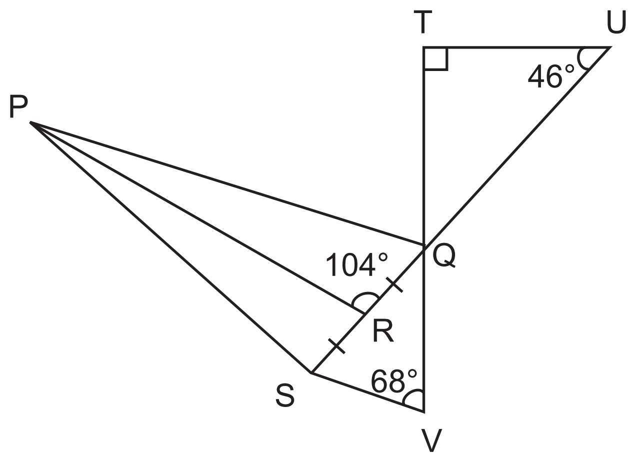
C. $\overline{WY}$ and $\overline{XZ}$ are perpendicular

D. $\angle WXY$ and $\angle XYZ$ are congruent

MODULE 1—Geometric Properties and Reasoning

Standard G.1.2.1.3

A diagram is shown below.



Which of the triangles **must** be isosceles?

A. △SPR

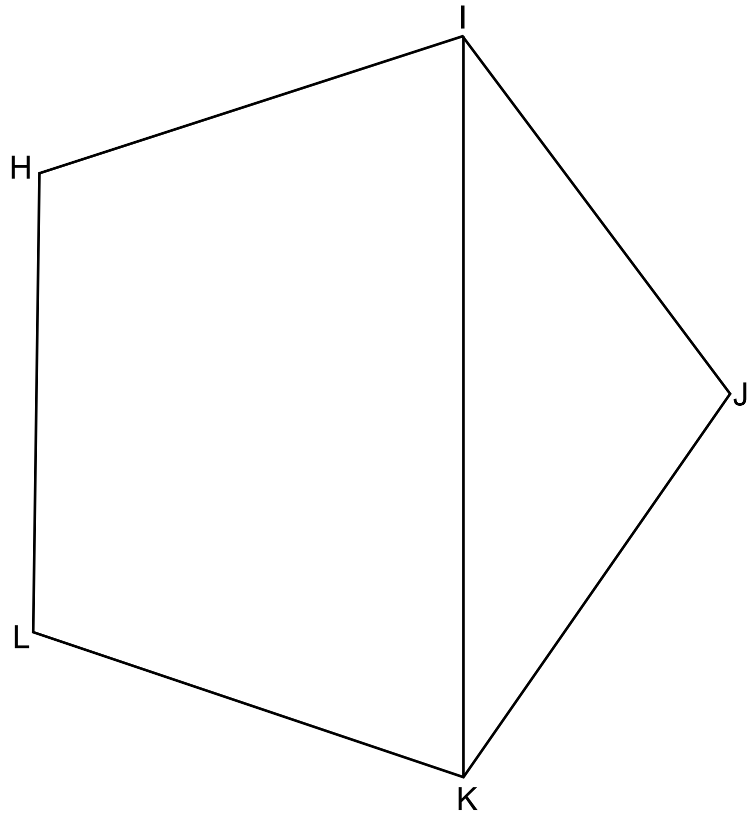
B. △SPQ

C. △QTU

D. △SQV

Standard G.1.2.1.4

Regular pentagon HIJKL is shown below.



What is the measure of $\angle$HIK?

A. 36°

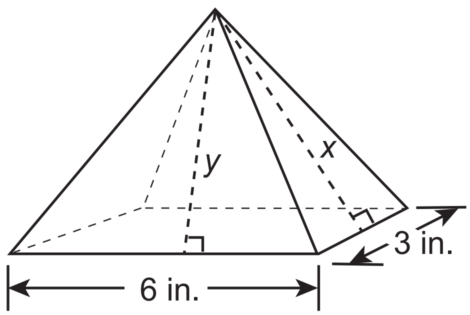
B. 54°

C. 72°

D. 108°

Standard G.1.2.1.5

In the right rectangular pyramid shown below, x and y are slant heights.



Which of the following **must** be true about the relationship between the values of x and y?

A. $x = y$

B. $x > y$

C. $x < y$

D. $x^2 + y^2 = 9^2$

ASSESSMENT ANCHOR
G.1.2 Properties of Polygons and Polyhedra

Sample Exam Questions

Standard G.1.2

A craftsman makes a cabinet in the shape of a triangular prism. The top and bottom of the cabinet are congruent isosceles right triangles.

A. Describe the shape needed to build each of the faces of the cabinet.

In order to make production of the cabinets easier, the craftsman wants to design the cabinet so the lateral faces are all congruent figures.

B. Explain why this is not possible.

Continued on next page.

Continued. Please refer to the previous page for task explanation.

C. What must be true about the base of a triangular prism in order for the lateral faces to all be congruent figures?

The craftsman is designing the cabinet to fit perfectly against two perpendicular walls.

D. Explain why a cabinet whose lateral faces are all squares cannot fit perfectly against two perpendicular walls.

Standard G.1.2

Kelly notices that in an equilateral triangle, each interior angle measures 60° and in a square, each interior angle measures 90°. She wonders if, each time a side is added to the number of sides in a regular polygon, the measure of each interior angle increases by 30°.

Kelly examines a regular pentagon to see if her hypothesis is correct.

A. What is the measure of each interior angle of a regular pentagon?

interior angle of a regular pentagon: ______________________

Kelly decides to examine the ratios of the measures of the interior angles to look for a pattern. She notices that the ratio of the measure of each interior angle of an equilateral triangle to a square is $\frac{2}{3}$.

B. What is the ratio of the measure of each interior angle of a square to a regular pentagon?

ratio: ______________________

Continued on next page.

Continued. Please refer to the previous page for task explanation.

Kelly makes a ratio comparing the measure of each interior angle of a regular polygon with n sides to the measure of each interior angle of a regular polygon with $n + 1$ sides.

C. What is the ratio?

ratio: ____________________

D. What is the ratio of the measure of each interior angle of a regular 9-sided polygon to the measure of each interior angle of a regular 10-sided polygon?

ratio: ____________________

ASSESSMENT ANCHOR G.1.3 Congruence, Similarity, and Proofs				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.1.3.1	Use properties of congruence, correspondence, and similarity in problem-solving settings involving two- and three-dimensional figures.	**G.1.3.1.1**	Identify and/or use properties of congruent and similar polygons or solids.	**CC.2.3.HS.A.1 CC.2.3.HS.A.2 CC.2.3.HS.A.5 CC.2.3.HS.A.6**
		G.1.3.1.2	Identify and/or use proportional relationships in similar figures.	

Sample Exam Questions

Standard G.1.3.1.1

Triangle CDE is similar to triangle FGH. Which relationship **must** be true?

A. $\frac{CD + CE}{DE} = \frac{GH + FH}{GH}$

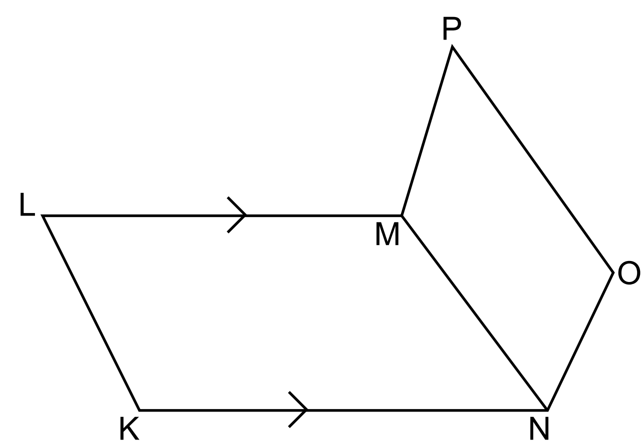
B. $\frac{CD}{CE} = \frac{FH}{FG}$

C. $m\angle ECD + m\angle CDE + m\angle GHF = 180°$

D. $m\angle ECD + m\angle CDE = m\angle HFG + m\angle GHF$

Standard G.1.3.1.2

Trapezoid KLMN is similar to trapezoid ONMP as shown below.



Which relationship **must** be true?

A. $\frac{MP}{KL} = \frac{ON}{MN}$

B. $\frac{LM}{PO} = \frac{NK}{NM}$

C. $\frac{NK}{PO} = \frac{NM}{LM}$

D. $\frac{ON}{KL} = \frac{MP}{MN}$

ASSESSMENT ANCHOR G.1.3 Congruence, Similarity, and Proofs		
Anchor Descriptor	**Eligible Content**	**PA Core Standards**
G.1.3.2 Write formal proofs and/or use logic statements to construct or validate arguments.	**G.1.3.2.1** Write, analyze, complete, or identify formal proofs (e.g., direct and/or indirect proofs/ proofs by contradiction).	CC.2.2.HS.C.9 CC.2.3.HS.A.3 CC.2.3.HS.A.6 CC.2.3.HS.A.8

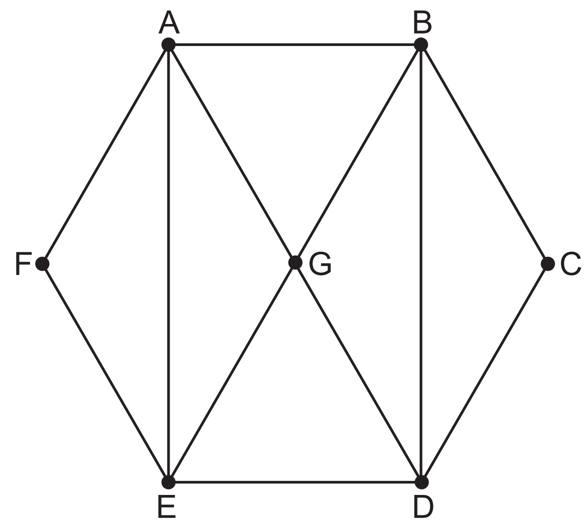
Sample Exam Question

Standard G.1.3.2.1

The diagram shown below is used in a proof.

Given: ABCDEF is a regular hexagon

Prove: $\triangle AED \cong \triangle DBA$



A proof is shown below, but statement 7 and reason 7 are missing.

Statements	Reasons
1. ABCDEF is a regular hexagon	1. Given
2. $\angle F \cong \angle C$	2. Angles of a regular polygon are congruent
3. $\overline{AF} \cong \overline{BC}$ $\overline{FE} \cong \overline{CD}$	3. Sides of a regular polygon are congruent
4. $\triangle AFE \cong \triangle BCD$	4. Side-angle-side congruence
5. $\overline{AE} \cong \overline{BD}$	5. Corresponding parts of congruent triangles are congruent
6. $\overline{ED} \cong \overline{AB}$	6. Sides of a regular polygon are congruent
7. ?	7. ?
8. $\triangle AED \cong \triangle DBA$	8. Side-side-side congruence

Continued on next page.

Standard G.1.3.2.1 *(continued)*

Which are **most likely** statement 7 and reason 7 that would complete the proof?

A. statement 7: $\angle AED \cong \angle ABD$
reason 7: Angles of a rectangle are congruent

B. statement 7: $\overline{AD} \cong \overline{BE}$
reason 7: Diagonals of a rectangle are congruent

C. statement 7: $\overline{DA} \cong \overline{AD}$
reason 7: Reflexive property of congruence

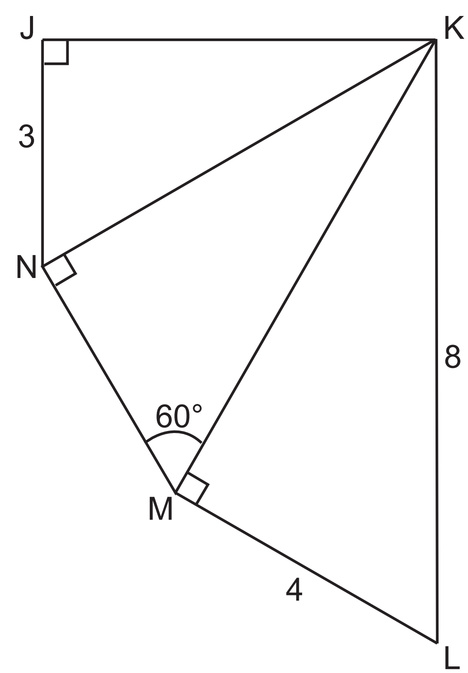
D. statement 7: $\overline{EG} \cong \overline{GB}$
reason 7: Diagonals of a rectangle bisect each other

ASSESSMENT ANCHOR
G.1.3 Congruence, Similarity, and Proofs

Sample Exam Questions

Standard G.1.3

In the diagram shown below, $\triangle JKN \sim \triangle NKM \sim \triangle MKL$.



A. What is the length, in units, of $\overline{NK}$?

length of $\overline{NK}$: ______________________ units

Continued on next page.

Continued. Please refer to the previous page for task explanation.

B. What is the length, in units, of $\overline{NM}$? Show your work. Explain your reasoning.

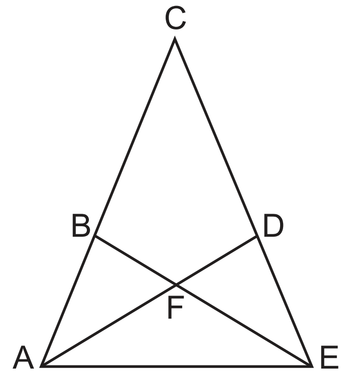
C. Prove that the measure of $\angle JKL$ is 90°.

Standard G.1.3

The diagram shown below is used in a proof.

Given: $\triangle ACE$ is isosceles
$\overline{AB} \cong \overline{ED}$

Prove: $\triangle ABE \cong \triangle EDA$



A proof is shown below, but statement 4 and reason 4 are missing.

Statements	Reasons
1. $\triangle ACE$ is isosceles	1. Given
2. $\angle EAC \cong \angle AEC$	2. Base angles of isosceles triangles are congruent
3. $\overline{AB} \cong \overline{ED}$	3. Given
4. ?	4. ?
5. $\triangle ABE \cong \triangle EDA$	5. Side-angle-side triangle congruence

A. What could be statement 4 and reason 4 to complete the proof?

statement 4: ______________________

reason for statement 4: ______________________

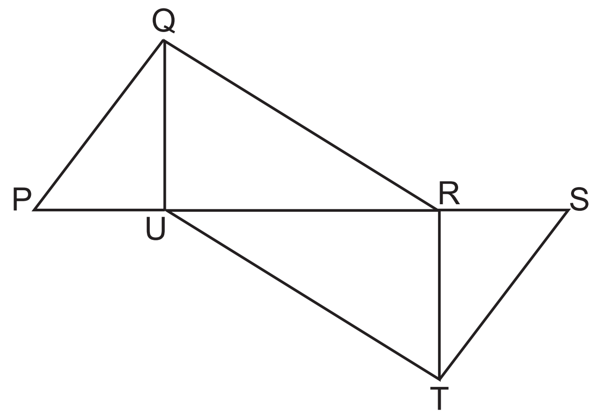
Continued on next page.

Continued. Please refer to the previous page for task explanation.

The diagram shown below is used in a proof.

Given: ?

Prove: $\triangle QUR \cong \triangle TRU$



A proof is shown below, but statements 1 and 3 are missing.

Statements	Reasons
1. ?	1. Given
2. $\overline{QR} \cong \overline{TU}$ $\angle QRP \cong \angle TUS$	2. Corresponding parts of congruent triangles are congruent
3. ?	3. Congruence of segments is reflexive
4. $\triangle QUR \cong \triangle TRU$	4. Side-angle-side triangle congruence

B. What could be statement 1 and statement 3 to complete the proof?

statement 1: ______________________________

statement 3: ______________________________

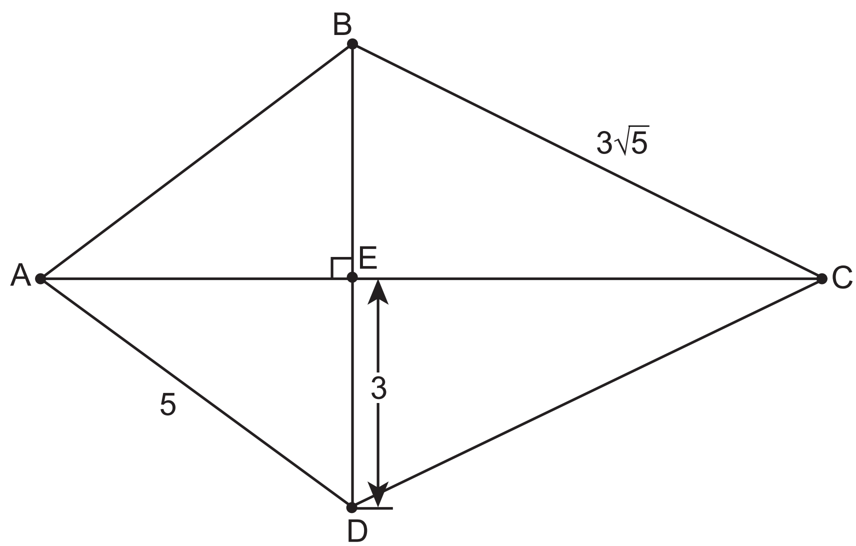
MODULE 2—Coordinate Geometry and Measurement

ASSESSMENT ANCHOR G.2.1 Coordinate Geometry and Right Triangles				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.2.1.1	Solve problems involving right triangles.	**G.2.1.1.1**	Use the Pythagorean theorem to write and/or solve problems involving right triangles.	**CC.2.2.HS.C.9** **CC.2.3.HS.A.7**
		G.2.1.1.2	Use trigonometric ratios to write and/or solve problems involving right triangles.	

Sample Exam Questions

Standard G.2.1.1.1

A kite is shown below.



What is the length of $\overline{AC}$?

A. 4

B. 6

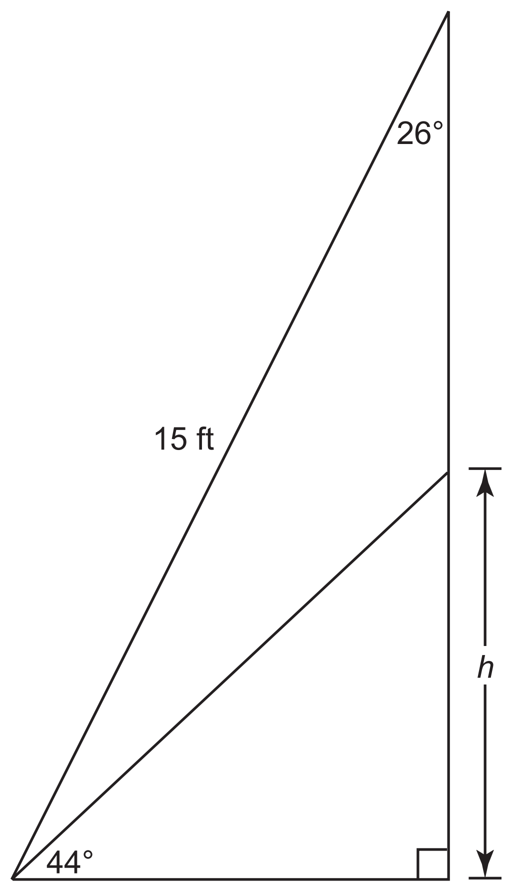
C. 9

D. 10

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.1.1.2

The hypotenuse of each right triangle shown below represents a ladder leaning against a building.



Which equation can be used to find *h*, the distance between the base of the building and the point where the shorter ladder touches the building?

A. $h = (\sin 44°)(15 \sin 26°)$

B. $h = (\sin 44°)(15 \cos 26°)$

C. $h = (\tan 26°)(15 \sin 44°)$

D. $h = (\tan 44°)(15 \sin 26°)$

MODULE 2—Coordinate Geometry and Measurement

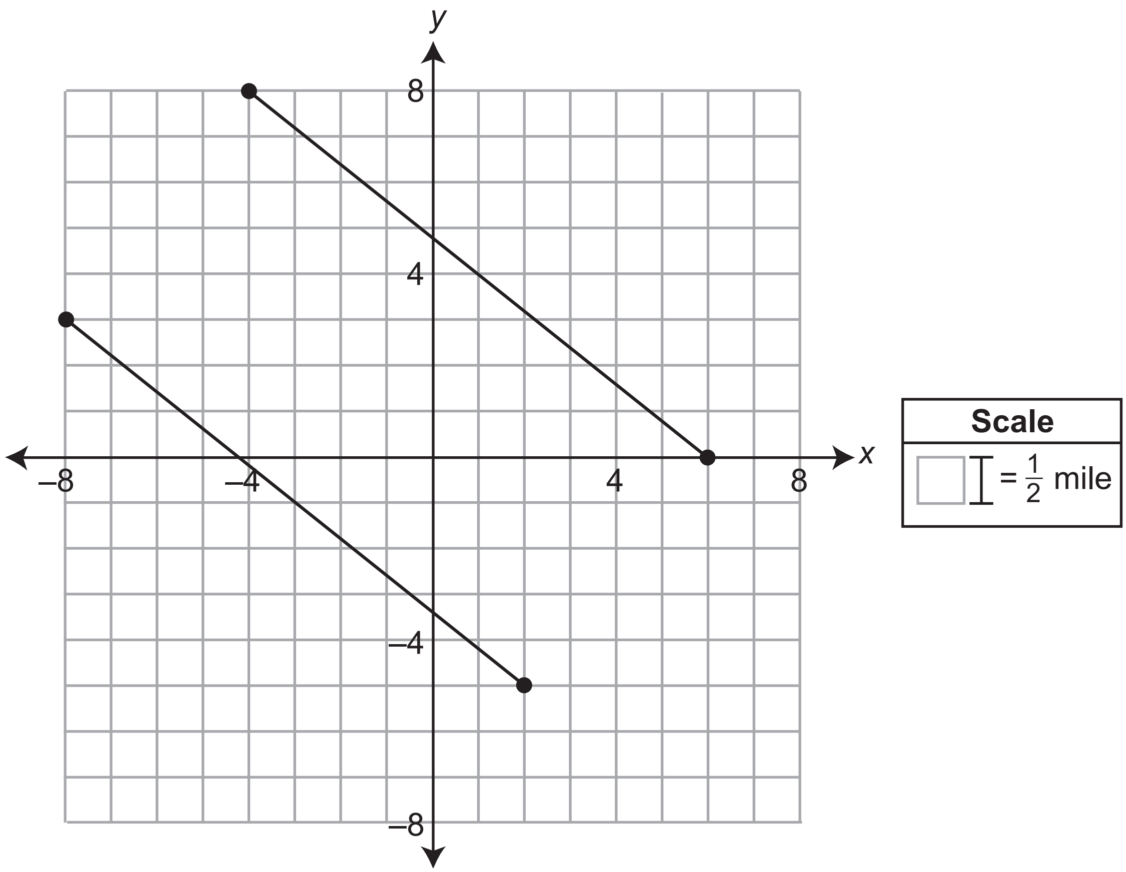
ASSESSMENT ANCHOR G.2.1 Coordinate Geometry and Right Triangles				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.2.1.2	Solve problems using analytic geometry.	**G.2.1.2.1**	Calculate the distance and/or midpoint between two points on a number line or on a coordinate plane.	CC.2.3.8.A.3 CC.2.3.HS.A.11
		G.2.1.2.2	Relate slope to perpendicularity and/or parallelism (limit to linear algebraic equations).	
		G.2.1.2.3	Use slope, distance, and/or midpoint between two points on a coordinate plane to establish properties of a two-dimensional shape.	

MODULE 2—Coordinate Geometry and Measurement

Sample Exam Questions

Standard G.2.1.2.1

The segments on the coordinate plane below represent two parallel roads in a neighborhood. Each unit on the plane represents $\frac{1}{2}$ mile.



A new road will be built connecting the midpoints of the two parallel roads. Which is the **closest approximation** of the length of the new road connecting the two midpoints?

A. 1.8 miles

B. 3.2 miles

C. 6.4 miles

D. 12.8 miles

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.1.2.2

Line *p* contains the points (9, 7) and (13, 5). Which equation represents a line perpendicular to line *p*?

A. $-2x + y = -11$

B. $-x - 2y = -2$

C. $-x + 2y = 5$

D. $2x + y = 31$

Standard G.2.1.2.3

A map shows flagpoles on a coordinate grid. The flagpoles are at (−5, 2), (−5, 6), (−1, 6), and (2, −1). What type of quadrilateral is formed by the flagpoles on the coordinate grid?

A. kite

B. rectangle

C. rhombus

D. square

MODULE 2—Coordinate Geometry and Measurement

ASSESSMENT ANCHOR
G.2.1 Coordinate Geometry and Right Triangles

Sample Exam Questions

Standard G.2.1

A scientist is plotting the circular path of a particle on a coordinate plane for a lab experiment. The scientist knows the path is a perfect circle and that the particle starts at ordered pair (−5, −11). When the particle is halfway around the circle, the particle is at the ordered pair (11, 19).

The segment formed by connecting these two points has the center of the circle as its midpoint.

A. What is the ordered pair that represents the center of the circle?

center of the circle: (________ , ________)

B. What is the length of the radius, in units, of the circle?

radius = ____________________ units

Continued on next page.

Continued. Please refer to the previous page for task explanation.

C. Explain why the particle can never pass through a point with an x-coordinate of 24 as long as it stays on the circular path.

The scientist knows the particle will intersect the line $y = 6$ twice. The intersection of the particle and the line can be expressed as the ordered pair $(x, 6)$. The value for the x-coordinate of one of the ordered pairs is $x \approx 19.88$.

D. State an approximate value for the **other** x-coordinate.

$x \approx$ ______________________

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.1

For a social studies project, Darius has to make a map of a neighborhood that could exist in his hometown. He wants to make a park in the shape of a right triangle. He has already planned 2 of the streets that make up 2 sides of his park. The hypotenuse of the park is 3rd Avenue, which goes through the points (−3, 2) and (9, 7) on his map.

One of the legs is Elm Street, which goes through (12, 5) and has a slope of $-\frac{2}{3}$. The other leg of the park will be Spring Parkway and will go through (−3, 2) and intersect Elm Street.

A. What is the slope of Spring Parkway?

slope = ______________________

B. What is the length, in units, of 3rd Avenue?

length = ______________________ units

Continued on next page.

MODULE 2—Coordinate Geometry and Measurement

Continued. Please refer to the previous page for task explanation.

The variable x represents the length of Spring Parkway in units. The measure of the angle formed by Spring Parkway and 3rd Avenue is approximately 33.69°.

C. Write a trigonometric equation relating the measure of the angle formed by Spring Parkway and 3rd Avenue, the length of Spring Parkway (x), and the length of 3rd Avenue from **part B**.

equation: ______________________________

D. Solve for x, the length of Spring Parkway in units.

x = ______________________________ units

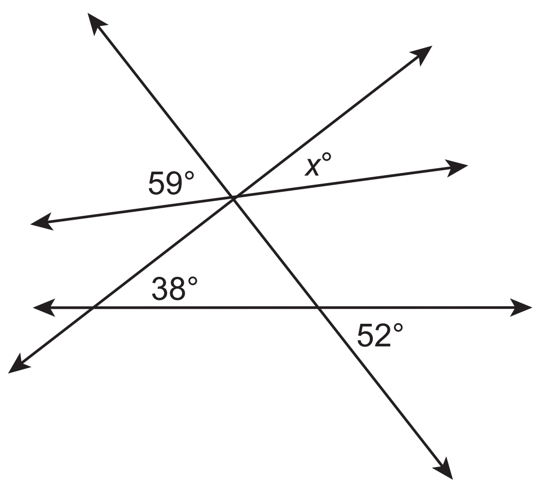
MODULE 2—Coordinate Geometry and Measurement

ASSESSMENT ANCHOR G.2.2 Measurements of Two-Dimensional Shapes and Figures				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.2.2.1	Use and/or compare measurements of angles.	**G.2.2.1.1**	Use properties of angles formed by intersecting lines to find the measures of missing angles.	CC.2.3.8.A.2 CC.2.3.HS.A.3
		G.2.2.1.2	Use properties of angles formed when two parallel lines are cut by a transversal to find the measures of missing angles.	

Sample Exam Questions

Standard G.2.2.1.1

A figure is shown below.



What is the value of *x*?

A. 14

B. 21

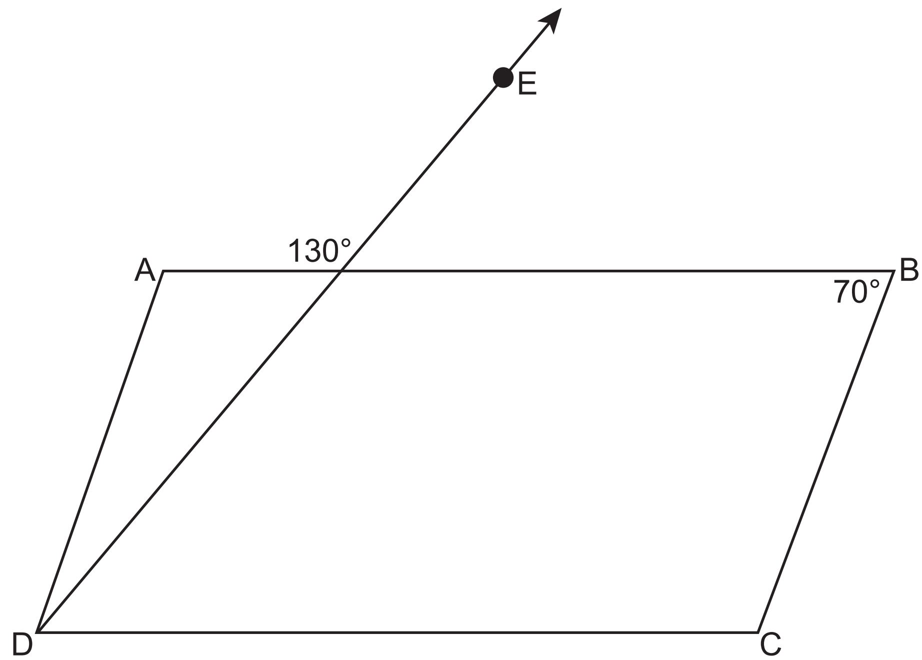
C. 31

D. 45

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.2.1.2

Parallelogram ABCD is shown below.



Ray DE passes through the vertex of ∠ADC. What is the measure of ∠ADE?

A. 20°

B. 40°

C. 50°

D. 70°

MODULE 2—Coordinate Geometry and Measurement

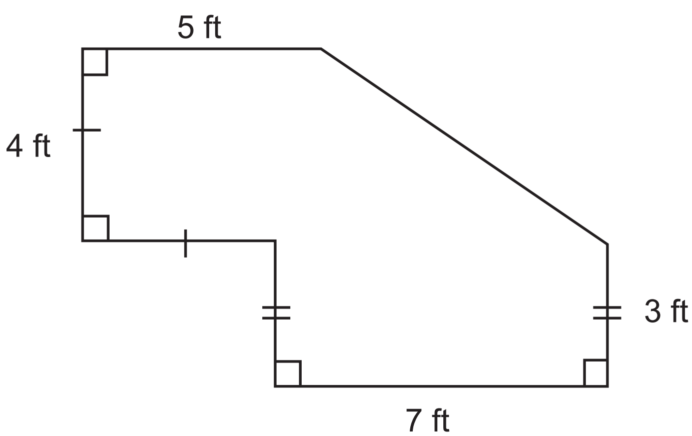
ASSESSMENT ANCHOR				
G.2.2	**Measurements of Two-Dimensional Shapes and Figures**			
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.2.2.2	Use and/or develop procedures to determine or describe measures of perimeter, circumference, and/or area. (May require conversions within the same system.)	**G.2.2.2.1**	Estimate area, perimeter, or circumference of an irregular figure.	**CC.2.2.HS.C.1** **CC.2.3.HS.A.3** **CC.2.3.HS.A.9**
		G.2.2.2.2	Find the measurement of a missing length, given the perimeter, circumference, or area.	
		G.2.2.2.3	Find the side lengths of a polygon with a given perimeter to maximize the area of the polygon.	
		G.2.2.2.4	Develop and/or use strategies to estimate the area of a compound/composite figure.	
		G.2.2.2.5	Find the area of a sector of a circle.	

Sample Exam Questions

Standard G.2.2.2.1

A diagram of Jacob's deck is shown below.

Jacob's Deck



Which is the **closest approximation** of the perimeter of the deck?

A. 29 feet

B. 30 feet

C. 33 feet

D. 34 feet

Standard G.2.2.2.2

A rectangular basketball court has a perimeter of 232 feet. The length of the court is 32 feet greater than the width. What is the width of the basketball court?

A. 42 feet

B. 74 feet

C. 84 feet

D. 100 feet

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.2.2.3

Stephen is making a rectangular garden. He has purchased 84 feet of fencing. What length (l) and width (w) will maximize the area of a garden with a perimeter of 84 feet?

A. l = 21 feet
   w = 21 feet

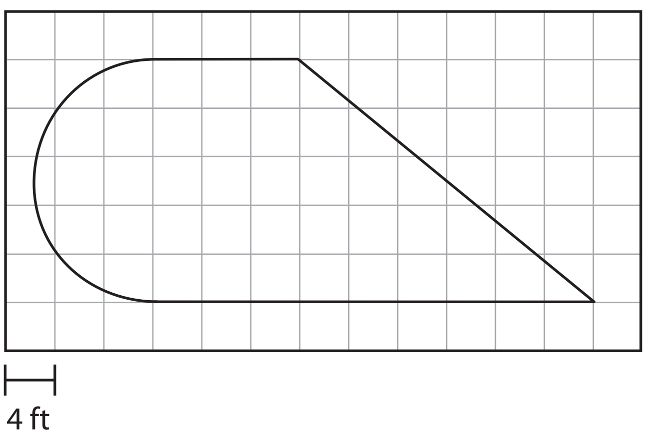
B. l = 32 feet
   w = 10 feet

C. l = 42 feet
   w = 42 feet

D. l = 64 feet
   w = 20 feet

Standard G.2.2.2.4

A figure is shown on the grid below.



Which expression represents the area of the figure?

A. $(\pi(2.5)^2 + (3 + 9)5)$ ft^2

B. $\left(\frac{1}{2}\pi(2.5)^2 + \frac{1}{2}(3 + 9)5\right)$ ft^2

C. $\left(\frac{1}{2}\pi(2.5)^2 + \frac{1}{2}(3 + 9)5\right)4$ ft^2

D. $\left(\frac{1}{2}\pi(2.5)^2 + \frac{1}{2}(3 + 9)5\right)(4 \bullet 4)$ ft^2

Standard G.2.2.2.5

A circular pizza with a diameter of 18 inches is cut into 8 equal-sized slices. Which is the **closest approximation** to the area of 1 slice of pizza?

A. 15.9 square inches

B. 31.8 square inches

C. 56.5 square inches

D. 127.2 square inches

MODULE 2—Coordinate Geometry and Measurement

ASSESSMENT ANCHOR G.2.2 Measurements of Two-Dimensional Shapes and Figures				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.2.2.3	Describe how a change in one dimension of a two-dimensional figure affects other measurements of that figure.	**G.2.2.3.1**	Describe how a change in the linear dimension of a figure affects its perimeter, circumference, and area (e.g., How does changing the length of the radius of a circle affect the circumference of the circle?).	**CC.2.3.HS.A.8** **CC.2.3.HS.A.9**

Sample Exam Question

Standard G.2.2.3.1

Jack drew a rectangle labeled X with a length of 10 centimeters (cm) and a width of 25 cm. He drew another rectangle labeled Y with a length of 15 cm and the same width as rectangle X. Which is a true statement about the rectangles?

A. The area of rectangle Y will be $\frac{3}{5}$ times greater than the area of rectangle X.

B. The area of rectangle Y will be $\frac{3}{2}$ times greater than the area of rectangle X.

C. The perimeter of rectangle Y will be $\frac{3}{5}$ times greater than the perimeter of rectangle X.

D. The perimeter of rectangle Y will be $\frac{3}{2}$ times greater than the perimeter of rectangle X.

MODULE 2—Coordinate Geometry and Measurement

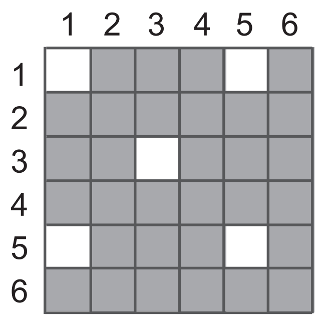
ASSESSMENT ANCHOR G.2.2 Measurements of Two-Dimensional Shapes and Figures		
Anchor Descriptor	**Eligible Content**	**PA Core Standards**
G.2.2.4 Apply probability to practical situations.	**G.2.2.4.1** Use area models to find probabilities.	**CC.2.3.HS.A.14**

Sample Exam Questions

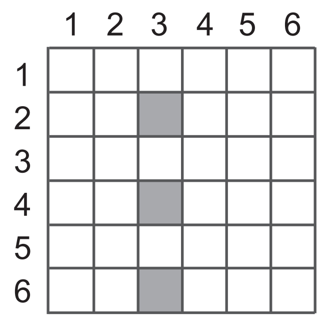
Standard G.2.2.4.1

Jamal rolls two 6-sided number cubes labeled 1 through 6. Which area model can Jamal use to correctly determine the probability that one of the number cubes will have a 3 facing up and the other will have an even number facing up?

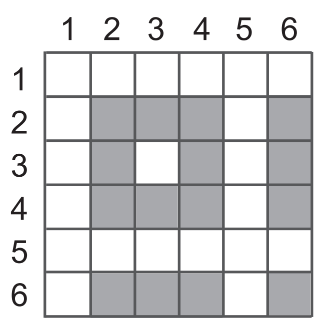
A.



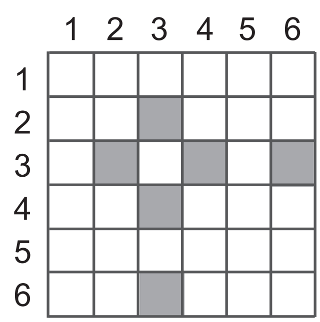
B.



C.



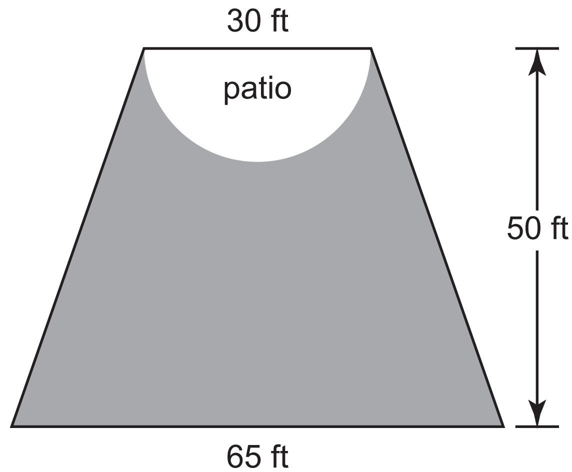
D.



MODULE 2—Coordinate Geometry and Measurement

Standard G.2.2.4.1

Michael's backyard is in the shape of an isosceles trapezoid and has a semicircular patio, as shown in the diagram below.



On a windy fall day, a leaf lands randomly in Michael's backyard. Which is the **closest approximation** of the probability that the leaf lands somewhere in the section of the backyard represented by the shaded region in the diagram?

A. 15%

B. 30%

C. 70%

D. 85%

MODULE 2—Coordinate Geometry and Measurement

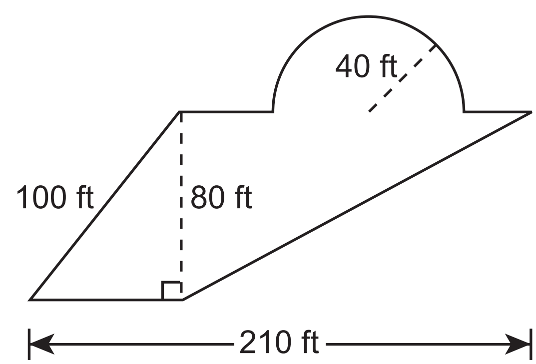
ASSESSMENT ANCHOR
G.2.2 Measurements of Two-Dimensional Shapes and Figures

Sample Exam Questions

Standard G.2.2

The diagram below shows the dimensions of Tessa's garden.

Tessa's Garden



A. What is the perimeter, in feet, of Tessa's garden? Show or explain all your work.

Continued on next page.

Continued. Please refer to the previous page for task explanation.

B. What is the area, in square feet, of Tessa's garden?

area of Tessa's garden: ______________________ square feet

Tessa decided that she liked the shape of her garden but wanted to have 2 times the area. She drew a design for a garden with every dimension multiplied by 2.

C. Explain the error in Tessa's design.

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.2

Donovan and Eric are playing in a checkers tournament. Donovan has a 70% chance of winning his game. Eric has a 40% chance of winning his game.

A. What is the difference between the probability that Donovan and Eric both win and the probability that they both lose?

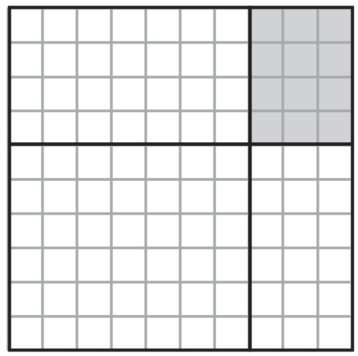
difference: ______________________________

Continued on next page.

MODULE 2—Coordinate Geometry and Measurement

Continued. Please refer to the previous page for task explanation.

Donovan and Eric created the probability model shown below to represent all possible outcomes for each of them playing 1 game.



B. Describe the compound event that is represented by the shaded region.

Sari and Keiko are also playing in a checkers tournament. The probability that Sari wins her game is double the probability that Keiko wins her game. The probability Sari wins her game **and** Keiko loses her game is 0.48.

C. What is the probability that Sari wins her game?

probability Sari wins: ______________________

MODULE 2—Coordinate Geometry and Measurement

ASSESSMENT ANCHOR				
G.2.3	**Measurements of Three-Dimensional Shapes and Figures**			
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.2.3.1	Use and/or develop procedures to determine or describe measures of surface area and/or volume. (May require conversions within the same system.)	**G.2.3.1.1**	Calculate the surface area of prisms, cylinders, cones, pyramids, and/or spheres. Formulas are provided on a reference sheet.	**CC.2.3.8.A.1** **CC.2.3.HS.A.12** **CC.2.3.HS.A.14**
		G.2.3.1.2	Calculate the volume of prisms, cylinders, cones, pyramids, and/or spheres. Formulas are provided on a reference sheet.	
		G.2.3.1.3	Find the measurement of a missing length, given the surface area or volume.	

Sample Exam Questions

Standard G.2.3.1.1

A sealed container is shaped like a right cylinder. The exterior height is 80 cm. The exterior diameter of each base is 28 cm. The circumference of each base is approximately 87.92 cm. The longest diagonal is approximately 84.76 cm. The measure of the total exterior surface area of the container can be used to determine the amount of paint needed to cover the container. Which is the **closest approximation** of the total exterior surface area, including the bases, of the container?

A. 7,209.44 square cm

B. 7,649.04 square cm

C. 8,011.68 square cm

D. 8,264.48 square cm

Standard G.2.3.1.2

A freshwater tank shaped like a rectangular prism has a length of 72 inches, a width of 24 inches, and a height of 25 inches. The tank is filled with water at a constant rate of 5 cubic feet per hour. How long will it take to fill the tank halfway?

A. 2.5 hours

B. 5 hours

C. 12.5 hours

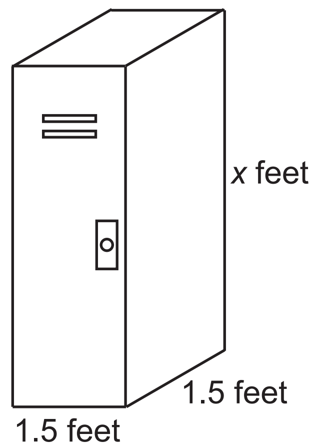
D. 25 hours

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.3.1.3

Every locker at a school has a volume of 10.125 cubic feet. The length and width are both 1.5 feet as shown below.

Locker



What is the value of x?

A. 3.375

B. 4.5

C. 6.75

D. 7.125

MODULE 2—Coordinate Geometry and Measurement

ASSESSMENT ANCHOR G.2.3 Measurements of Three-Dimensional Shapes and Figures				
Anchor Descriptor		**Eligible Content**		**PA Core Standards**
G.2.3.2	Describe how a change in one dimension of a three-dimensional figure affects other measurements of that figure.	**G.2.3.2.1**	Describe how a change in the linear dimension of a figure affects its surface area or volume (e.g., How does changing the length of the edge of a cube affect the volume of the cube?).	**CC.2.3.HS.A.13**

Sample Exam Question

Standard G.2.3.2.1

Anya is wrapping gift boxes in paper. Each gift box is a rectangular prism. The larger box has a length, width, and height twice as large as the smaller box. Which statement shows the relationship between the surface area of the gift boxes?

A. The larger gift box has a surface area 2 times as large as the smaller gift box.

B. The larger gift box has a surface area 4 times as large as the smaller gift box.

C. The larger gift box has a surface area 6 times as large as the smaller gift box.

D. The larger gift box has a surface area 8 times as large as the smaller gift box.

MODULE 2—Coordinate Geometry and Measurement

ASSESSMENT ANCHOR
G.2.3 Measurements of Three-Dimensional Shapes and Figures

Sample Exam Questions

Standard G.2.3

Max is building a spherical model of Earth. He is building his model using 2-inch-long pieces of wood to construct the radius.

The first time he tries to build the model, he uses 3 of the 2-inch pieces of wood end-to-end to make the radius of the model.

A. What is the volume of the model in cubic inches?

volume: ______________________ cubic inches

Continued on next page.

MODULE 2—Coordinate Geometry and Measurement

Continued. Please refer to the previous page for task explanation.

In order to purchase the right amount of paint for the outside of the model, Max needs to know the surface area.

B. What is the surface area of the model in square inches?

surface area: ______________________ square inches

Max decides he wants the model to be larger. He wants the new model to have exactly twice the volume of the original model.

C. Explain why Max cannot make a model that has exactly twice the volume without breaking the 2-inch-long pieces of wood he is using to construct the radius.

Max is going to make a new larger model using *n* 2-inch-long pieces of wood.

D. How many times greater than the surface area of the original model will the new model be?

MODULE 2—Coordinate Geometry and Measurement

Standard G.2.3

An engineer for a storage company is designing cylindrical containers.

A. The first container he designs is a cylinder with a radius of 3 inches and a height of 8 inches. What is the volume of the container in cubic inches?

volume: ______________________ cubic inches

B. When the surface area is x square inches and the volume is x cubic inches, what is the measure of the height (h) in terms of the radius (r)?

$h =$ ______________________

Continued on next page.

MODULE 2—Coordinate Geometry and Measurement

Continued. Please refer to the previous page for task explanation.

A cylindrical container has a surface area of x square inches and a volume of x cubic inches. The radius of the cylindrical container is 6 inches.

C. Using your equation from **part B**, what is the height, in inches, of the cylindrical container?

height: ______________________ inches

D. What number (x) represents both the surface area, in square inches, and the volume, in cubic inches, of the cylindrical container?

x = ______________________

KEYSTONE GEOMETRY ASSESSMENT ANCHORS
KEY TO SAMPLE MULTIPLE-CHOICE ITEMS

Geometry

Eligible Content	Key
G.1.1.1.1	C
G.1.1.1.2	D
G.1.1.1.3	C
G.1.1.1.4	A

Eligible Content	Key
G.1.2.1.1	C
G.1.2.1.2	C
G.1.2.1.3	D
G.1.2.1.4	C
G.1.2.1.5	B

Eligible Content	Key
G.1.3.1.1	C
G.1.3.1.2	D
G.1.3.2.1	C

Eligible Content	Key
G.2.1.1.1	D
G.2.1.1.2	D

Eligible Content	Key
G.2.1.2.1	B
G.2.1.2.2	A
G.2.1.2.3	A

Eligible Content	Key
G.2.2.1.1	C
G.2.2.1.2	A

Eligible Content	Key
G.2.2.2.1	C
G.2.2.2.2	A
G.2.2.2.3	A
G.2.2.2.4	D
G.2.2.2.5	B

Eligible Content	Key
G.2.2.3.1	B

Eligible Content	Key
G.2.2.4.1 (page 42)	D
G.2.2.4.1 (page 43)	D

Eligible Content	Key
G.2.3.1.1	D
G.2.3.1.2	A
G.2.3.1.3	B

Eligible Content	Key
G.2.3.2.1	B

Keystone Exams: Geometry

Glossary to the
Assessment Anchor & Eligible Content

The Keystone Glossary includes terms and definitions associated with the Keystone Assessment Anchors and Eligible Content. The terms and definitions included in the glossary are intended to assist Pennsylvania educators in better understanding the Keystone Assessment Anchors and Eligible Content. The glossary does not define all possible terms included on an actual Keystone Exam, and it is not intended to define terms for use in classroom instruction for a particular grade level or course.

Pennsylvania Department of Education

www.education.state.pa.us

April 2014

Acute Angle
An angle that measures greater than 0° but less than 90°. An angle larger than a zero angle but smaller than a right angle.

Acute Triangle
A triangle in which each angle measures less than 90° (i.e., there are three acute angles).

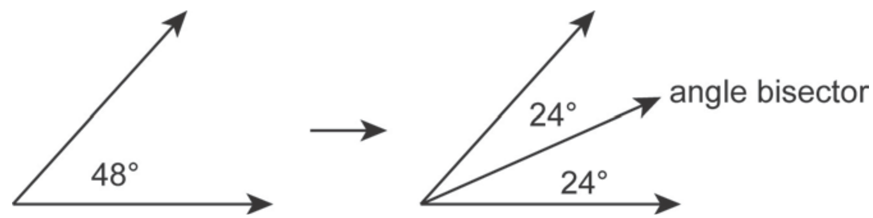
Altitude (of a Solid)
The shortest line segment between the base and the opposite vertex of a pyramid or cone, with one endpoint at the vertex. The shortest line segment between two bases of a prism or cylinder. The line segment is perpendicular to the base(s) of the solid. The altitude may extend from either the base of the solid or from the plane extending through the base. In a right solid, the altitude can be formed at the center of the base(s).

Altitude (of a Triangle)
A line segment with one endpoint at a vertex of the triangle that is perpendicular to the side opposite the vertex. The other endpoint of the altitude may either be on the side of the triangle or on the line extending through the side.

Analytic Geometry
The study of geometry using algebra (i.e., points, lines, and shapes are described in terms of their coordinates, then algebra is used to prove things about these points, lines, and shapes). The description of geometric figures and their relationships with algebraic equations or vice-versa.

Angle
The inclination between intersecting lines, line segments, and/or rays measured in degrees (e.g., a 90° inclination is a right angle). The figure is often represented by two rays that have a common endpoint.

Angle Bisector A line, line segment, or ray which cuts a given angle in half creating two congruent angles. Example:



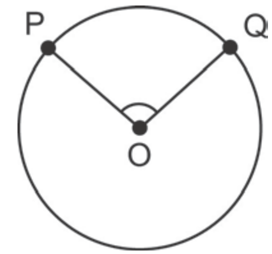
Arc (of a Circle) Any continuous part of a circle between two points on the circle.

Area The measure, in square units or units^2, of the surface of a plane figure (i.e., the number of square units it takes to cover the figure).

Base (Three Dimensions) In a cone or pyramid, the face of the figure which is opposite the vertex. In a cylinder or prism, either of the two faces of the figure which are parallel and congruent.

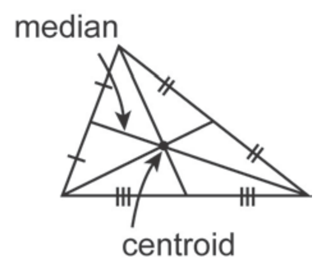
Base (Two Dimensions) In an isosceles triangle, the side of the figure which is adjacent to the congruent angles. In a trapezoid, either of the parallel sides of the figure.

Central Angle (of a Circle)	An angle whose vertex is at the center of a circle and whose sides are radii of that circle. Example:

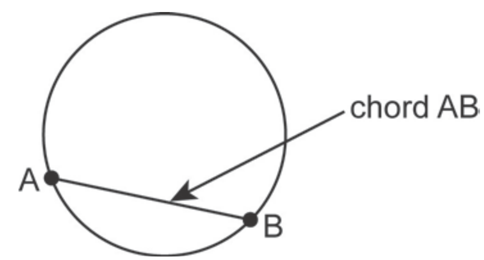


$\angle$ POQ with vertex O

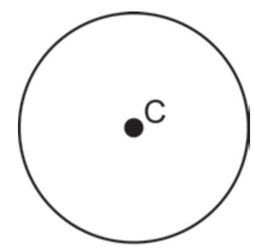
Central Angle (of a Regular Polygon)	An angle whose vertex is at the center of the polygon and whose sides intersect the regular polygon at adjacent vertices.
Centroid	A point of concurrency for a triangle that can be found at the intersection of the three medians of the triangle. This point is also the center of balance of a triangle with uniform mass. It is sometimes referred to as the "center of gravity." Example:



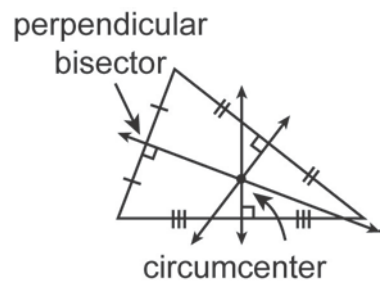
Chord A line segment whose two endpoints are on the perimeter of a circle. A particular type of chord that passes through the center of the circle is called a diameter. A chord is part of a secant of the circle. Example:



Circle A two-dimensional figure for which all points are the same distance from its center. Informally, a perfectly round shape. The circle is named for its center point. Example:



Circumcenter A point of concurrency for a triangle that can be found at the intersection of the three perpendicular bisectors of the triangle. This point is also the center of a circle that can be circumscribed about the triangle. Example:



Circumference (of a Circle) The total measured distance around the outside of a circle. The circle's perimeter. More formally, a complete circular arc.

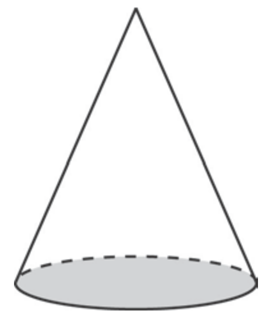
Circumscribed Circle A circle around a polygon such that each vertex of the polygon is a point on the circle.

Colinear Two or more points that lie on the same line.

Composite (Compound) Figure (Shape) A figure made from two or more geometric figures (i.e., from "simpler" figures).

Cone

A three-dimensional figure with a single circular base and one vertex. A curved surface connects the base and the vertex. The shortest distance from the base to the vertex is called the altitude. If the altitude goes through the center of the base, the cone is called a “right cone”; otherwise, it is called an “oblique cone.” Unless otherwise specified, it may be assumed all cones are right cones. Example:



cone

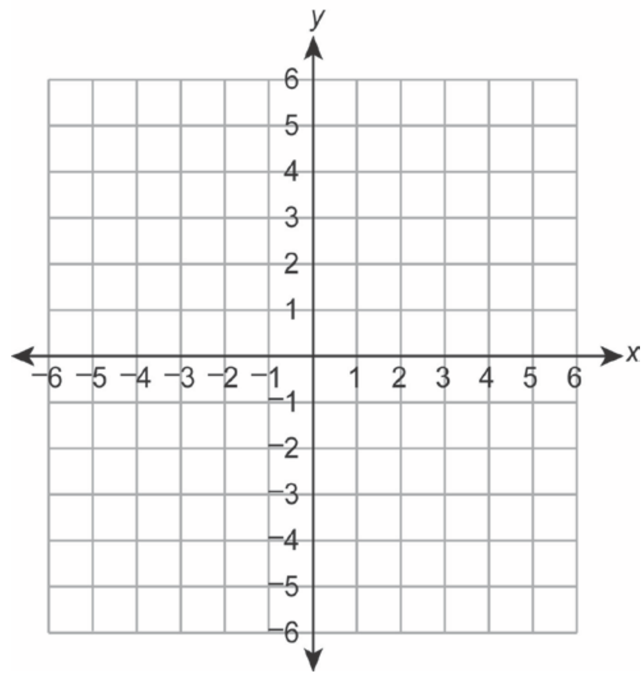
Congruent Figures

Two or more figures having the same shape and size (i.e., measure). Angles are congruent if they have the same measure. Line segments are congruent if they have the same length. Two or more shapes or solids are said to be congruent if they are “identical” in every way except for possibly their position. When congruent figures are named, their corresponding vertices are listed in the same order (e.g., if triangle ABC is congruent to triangle XYZ, then vertex C corresponds to vertex Z).

Conversion

The process of changing the form of a measurement, but not its value (e.g., 4 inches converts to $\frac{1}{3}$ foot; 4 square meters converts to 0.000004 square kilometers; 4 cubic feet converts to 6,912 cubic inches).

Coordinate Plane A plane formed by perpendicular number lines. The horizontal number line is the *x*-axis, and the vertical number line is the *y*-axis. The point where the axes meet is called the origin. Example:



coordinate plane

Coordinates The ordered pair of numbers giving the location or position of a point on a coordinate plane. The ordered pairs are written in parentheses (e.g., (*x*, *y*) where the *x*-coordinate is the first number in an ordered pair and represents the horizontal position of an object in a coordinate plane and the *y*-coordinate is the second number in an ordered pair and represents the vertical position of an object in a coordinate plane).

Coplanar Two or more figures that lie in the same plane.

Corresponding Angles Pairs of angles having the same relative position in geometric figures (i.e., angles on the same side of a transversal formed when two parallel lines are intersected by the transversal; four such pairs are formed, and the angles within the pairs are equal to each other). Corresponding angles are equal in measure.

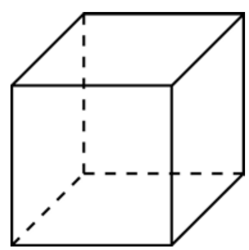
Corresponding Parts Two parts (angles, sides, or vertices) having the same relative position in congruent or similar figures. When congruent or similar figures are named, their corresponding vertices are listed in the same order (e.g., if triangle ABC is similar to triangle XYZ, then vertex C corresponds to vertex Z). See also corresponding angles and corresponding sides.

Corresponding Sides Two sides having the same relative position in two different figures. If the figures are congruent or similar, the sides may be, respectively, equal in length or proportional.

Cosine (of an Angle) A trigonometric ratio within a right triangle. The ratio is the length of the leg adjacent to the angle to the length of the hypotenuse of the triangle.

$$\text{cosine of an angle} = \frac{\text{length of adjacent leg}}{\text{length of hypotenuse}}$$

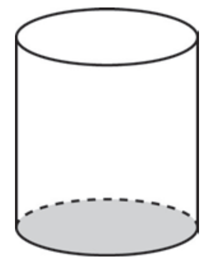
Cube A three-dimensional figure (e.g., a rectangular solid or prism) having six congruent square faces. Example:



cube

Cylinder

A three-dimensional figure with two circular bases that are parallel and congruent and joined by straight lines creating a lateral surface that is curved. The distance between the bases is called an altitude. If the altitude goes through the center of the bases, the cylinder is called a “right cylinder”; otherwise, it is called an “oblique cylinder.” Unless otherwise specified, it may be assumed all cylinders are right cylinders. Example:



cylinder

Degree

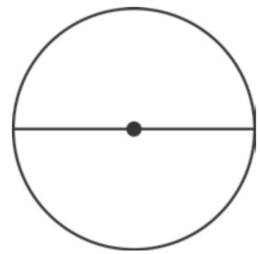
A unit of angle measure equal to $\frac{1}{360}$ of a complete revolution. There are 360 degrees in a circle. The symbol for degree is ° (e.g., 45° is read “45 degrees”).

Diagonal

Any line segment, other than a side or edge, within a polygon or polyhedron that connects one vertex with another vertex.

Diameter (of a Circle)

A line segment that has endpoints on a circle and passes through the center of the circle. It is the longest chord in a circle. It divides the circle in half. Example:



Direct Proof

The truth or validity of a given statement shown by a straightforward combination of established facts (e.g., existing axioms, definitions, theorems), without making any further assumptions (i.e., a sequence of statements showing that if one thing is true, then something following from it is also true).

Distance between Two Points

The space showing how far apart two points are (i.e., the shortest length between them).

Edge

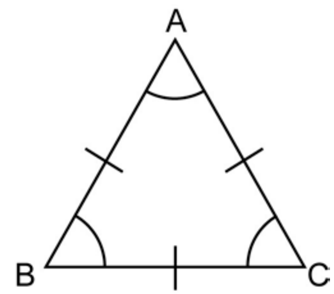
The line segment where two faces of a polyhedron meet (e.g., a rectangular prism has 12 edges). The endpoints of an edge are vertices of the polyhedron.

Endpoint

A point that marks the beginning or the end of a line segment; a point that marks the beginning of a ray.

Equilateral Triangle

A triangle where all sides are the same length (i.e., the sides are congruent). Each of the angles in an equilateral triangle is 60°. Thus, the triangle is also "equiangular." Example:



equilateral triangle ABC

Exterior Angle

An angle formed by a side of a polygon and an extension of an adjacent side. The measure of the exterior angle is supplementary to the measure of the interior angle at that vertex.

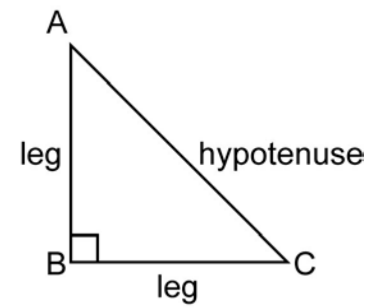
Face

A plane figure or flat surface that makes up one side of a three-dimensional figure or solid figure. Two faces meet at an edge, three or more faces meet at a vertex (e.g., a cube has 6 faces). See also lateral face.

Figure

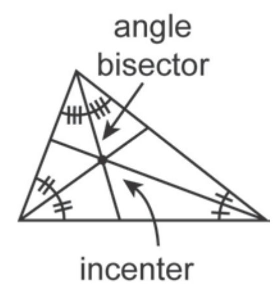
Any combination of points, lines, rays, line segments, angles, planes, or curves in two or three dimensions. Formally, it is any set of points on a plane or in space.

Hypotenuse The longest side of a right triangle (i.e., the side always opposite the right angle). Example:



right triangle ABC, with hypotenuse AC

Incenter A point of concurrency for a triangle that can be found at the intersection of the three angle bisectors of the triangle. This point is also the center of a circle that can be inscribed within the triangle. Example:



Indirect Proof A set of statements in which a false assumption is made. Using true or valid arguments, a statement is arrived at, but it is clearly wrong, so the original assumption must have been wrong. See also proof by contradiction.

Inscribed Circle A circle within a polygon such that each side of the polygon is tangent to the circle.

Interior Angle

An angle formed by two adjacent sides of a polygon. The common endpoint of the sides form the vertex of the angle, with the inclination of measure being on the inside of the polygon.

Intersecting Lines

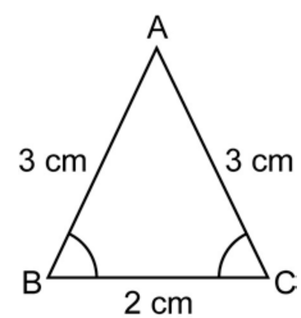
Two lines that cross or meet each other. They are coplanar, have only one point in common, have slopes that are not equal, are not parallel, and form angles at the point of intersection.

Irregular Figure

A figure that is not regular; not all sides and/or angles are congruent.

Isosceles Triangle

A triangle that has at least two congruent sides. The third side is called the base. The angles opposite the equal sides are also congruent. Example:



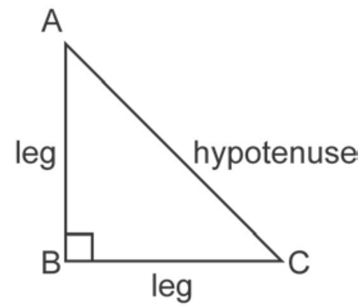
isosceles triangle ABC, with base BC

Lateral Face

Any face or surface of a three-dimensional figure or solid that is not a base.

Leg (of a Right Triangle)

Either of the two sides that form a right angle in a right triangle. It is one of the two shorter sides of the triangle and always opposite an acute angle. It is not the hypotenuse. Example:



right triangle ABC, with legs AB and BC

Line

A figure with only one dimension—length (no width or height). A straight path extending in both directions with no endpoints. It is considered "never ending." Formally, it is an infinite set of connected points (i.e., a set of points so closely set down there are no gaps or spaces between them). The line AB is written $\overleftrightarrow{AB}$, where *A* and *B* are two points through which the line passes. Example:

line AB ($\overleftrightarrow{AB}$)

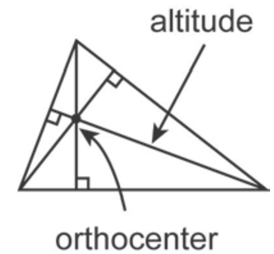
Line Segment

A part or piece of a line or ray with two fixed endpoints. Formally, it is the two endpoints and all points between them. The line segment AB is written $\overline{AB}$, where *A* and *B* are the endpoints of the line segment. Example:

line segment AB ($\overline{AB}$)

Linear Measurement	A measurement taken in a straight line.
Logic Statement (Proposition)	A statement examined for its truthfulness (i.e., proved true or false).
Median (of a Triangle)	A line segment with one endpoint at the vertex of a triangle and the other endpoint at the midpoint of the side opposite the vertex.
Midpoint	The point half-way between two given points (i.e., it divides or splits a line segment into two congruent line segments).
Obtuse Angle	An angle that measures more than 90° but less than 180°. An angle larger than a right angle but smaller than a straight angle.
Obtuse Triangle	A triangle with one angle that measures more than 90° (i.e., it has one obtuse angle and two acute angles).
Ordered Pair	A pair of numbers, (x, y), written in a particular order that indicates the position of a point on a coordinate plane. The first number, x, represents the x-coordinate and is the number of units left or right from the origin; the second number, y, represents the y-coordinate and is the number of units up or down from the origin.
Origin	The point (0, 0) on a coordinate plane. It is the point of intersection for the x-axis and the y-axis.

Orthocenter A point of concurrency for a triangle that can be found at the intersection of the three altitudes of the triangle. Example:



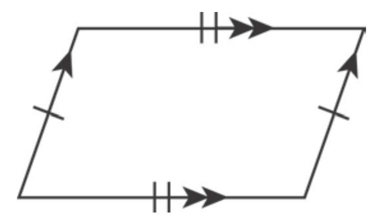
Parallel (Bases) Two bases of a three-dimensional figure that lie in parallel planes. All altitudes between the bases are congruent.

Parallel (Lines or Line Segments) Two distinct lines that are in the same plane and never intersect. On a coordinate grid, the lines have the same slope but different *y*-intercepts. They are always the same distance apart from each other. Parallel line segments are segments of parallel lines. The symbol for parallel is || (e.g., $\overline{AB} \parallel \overline{CD}$ is read "line segment AB is parallel to line segment CD").

Parallel (Planes) Two distinct planes that never intersect and are always the same distance apart.

Parallel (Sides) Two sides of a two-dimensional figure that lie on parallel lines.

Parallelogram A quadrilateral whose opposite sides are parallel and congruent (i.e., there are two pairs of parallel sides). Often one pair of these opposite sides is longer than the other pair. Opposite angles are also congruent, and the diagonals bisect each other. Example:


parallelogram

Perimeter The total distance around a closed figure. For a polygon, it is the sum of the lengths of its sides.

Perpendicular Two lines, segments, or rays that intersect, cross, or meet to form a 90° or right angle. The product of their slopes is $^-1$ (i.e., their slopes are "negative reciprocals" of each other). The symbol for perpendicular is $\perp$ (e.g., $\overline{AB} \perp \overline{CD}$ is read "line segment AB is perpendicular to line segment CD"). By definition, the two legs of a right triangle are perpendicular to each other.

Perpendicular Bisector A line that intersects a line segment at its midpoint and at a right angle.

π (Pi) The ratio of the circumference of a circle to its diameter. It is 3.14159265… to 1 or simply the value 3.14159265…. It can also be used to relate the radius of a circle to the circle's area. It is often approximated using either 3.14 or $\frac{22}{7}$.

Plane A set of points that forms a flat surface that extends infinitely in all directions. It has no height.

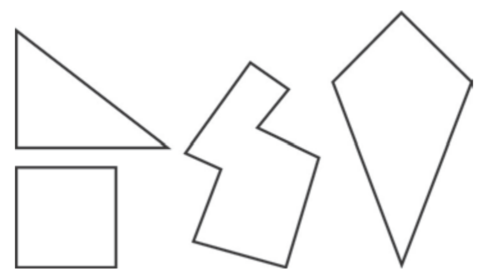
Plotting Points To place points on a coordinate plane using the *x*-coordinates and *y*-coordinates of the given points.

Point A figure with no dimensions—it has no length, width, or height. It is generally indicated with a single dot and is labeled with a single letter or an ordered pair on a coordinate plane. Example:

●P

point P

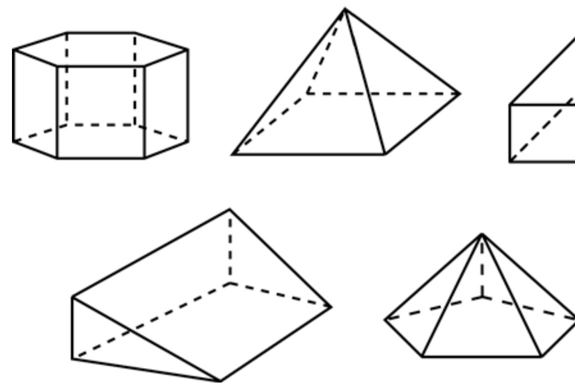
Polygon A closed plane figure made up of three or more line segments (i.e., a union of line segments connected end to end such that each segment intersects exactly two others at its endpoints); less formally, a flat shape with straight sides. The name of a polygon describes the number of sides/angles (e.g., triangle has three sides/angles, a quadrilateral has four, a pentagon has five, etc.). Examples:



polygons

Polyhedron

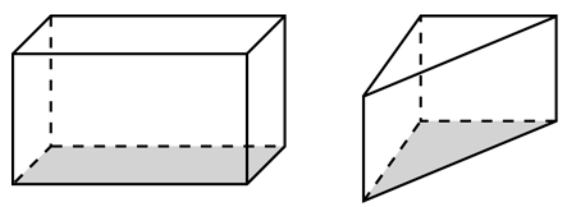
A three-dimensional figure or solid whose flat faces are all polygons where all edges are line segments. It has no curved surfaces or edges. The plural is "polyhedra." Examples:



polyhedra

Prism

A three-dimensional figure or polyhedron that has two congruent and parallel faces that are polygons called bases. The remaining faces, called lateral faces, are parallelograms (often rectangles). If the lateral faces are rectangles, the prism is called a "right prism"; otherwise, it is called an "oblique prism." Unless otherwise specified, it may be assumed all prisms are right prisms. Prisms are named by the shape of their bases. Examples:

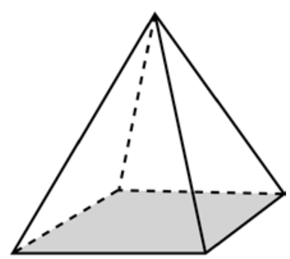


rectangular prism triangular prism

Proof by Contradiction A set of statements used to determine the truth of a proposition by showing that the proposition being untrue would imply a contradiction (i.e., one assumes that what is true is not true, then, eventually one discovers something that is clearly not true; when something is not not-true, then it is true). It is sometimes called the "law of double negation."

Proportional Relationship A relationship between two equal ratios. It is often used in problem-solving situations involving similar figures.

Pyramid A three-dimensional figure or polyhedron with a single polygon base and triangular faces that meet at a single point or vertex. The faces that meet at the vertex are called lateral faces. There is the same number of lateral faces as there are sides of the base. The shortest distance from the base to the vertex is called the altitude. If the altitude goes through the center of the base, the pyramid is called a "right pyramid"; otherwise, it is called an "oblique pyramid." Unless otherwise specified, it may be assumed all pyramids are right pyramids. A pyramid is named for the shape of its base (e.g., triangular pyramid or square pyramid). Example:

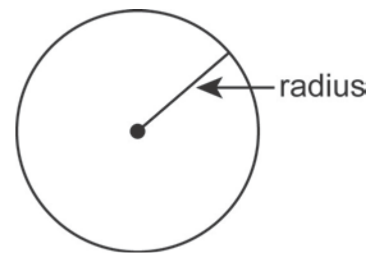


square pyramid

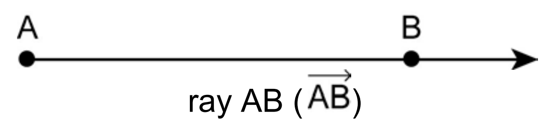
Pythagorean Theorem A formula for finding the length of a side of a right triangle when the lengths of two sides are given. It is $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.

Quadrilateral A four-sided polygon. It can be regular or irregular. The measures of its four interior angles always add up to 360°.

Radius (of a Circle) A line segment that has one endpoint at the center of the circle and the other endpoint on the circle. It is the shortest distance from the center of a circle to any point on the circle. It is half the length of the diameter. The plural is "radii." Example:

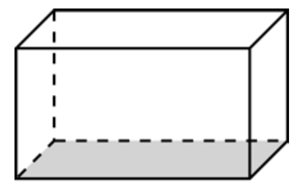


Ray A part or piece of a line with one fixed endpoint. Formally, it is the endpoint and all points in one direction. The ray AB is written $\overrightarrow{AB}$, where A is an endpoint of the ray that passes through point B. Example:



Rectangular Prism

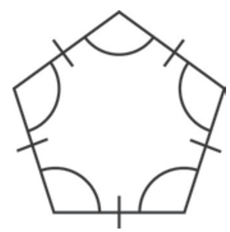
A three-dimensional figure or polyhedron which has two congruent and parallel rectangular bases. Informally, it is a "box shape" in three dimensions. Example:



rectangular prism

Regular Polygon

A polygon with sides all the same length and angles all the same size (i.e., all sides are congruent or equilateral, and all angles are congruent or equiangular). Example:

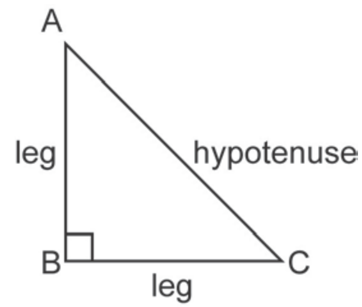


regular polygon

Right Angle

An angle that measures exactly 90°.

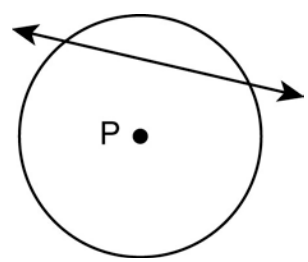
Right Triangle A triangle with one angle that measures 90° (i.e., it has one right angle and two acute angles). The side opposite the right angle is called the hypotenuse and the two other sides are called the legs.



right triangle ABC

Scalene Triangle A triangle that has no congruent sides (i.e., the three sides all have different lengths). The triangle also has no congruent angles (i.e., the three angles all have different measures).

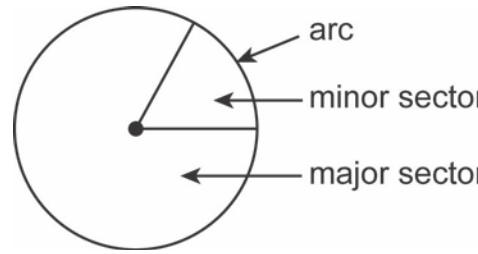
Secant (of a Circle) A line, line segment, or ray that passes through a circle at exactly two points. The segment of the secant connecting the points of intersection is a chord of the circle. Example:



secant

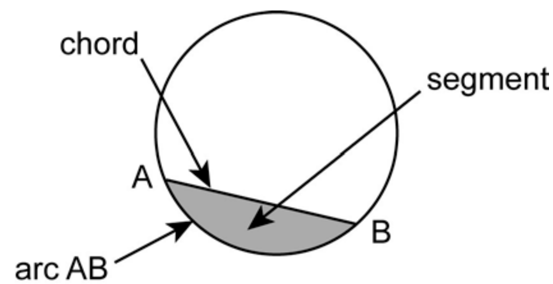
Sector (of a Circle)

The area or region between an arc and two radii at either end of that arc. The two radii divide or split the circle into two sectors called a "major sector" and a "minor sector." The major sector has a central angle of more than 180°, whereas the minor sector has a central angle of less than 180°. It is shaped like a slice of pie. Example:



Segment (of a Circle)

The area or region between an arc and a chord of a circle. Informally, the area of a circle "cut off" from the rest by a secant or chord. Example:



Semicircle

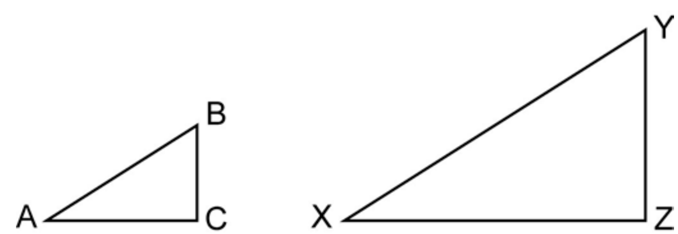
A half of a circle. A 180° arc. Formally, an arc whose endpoints lie on the diameter of the circle.

Shape

See figure.

Side — One of the line segments which make a polygon (e.g., a pentagon has five sides). The endpoints of a side are vertices of the polygon.

Similar Figures — Figures having the same shape, but not necessarily the same size. Often, one figure is the dilation ("enlargement") of the other. Formally, their corresponding sides are in proportion and their corresponding angles are congruent. When similar figures are named, their corresponding vertices are listed in the same order (e.g., if triangle ABC is similar to triangle XYZ, then vertex C corresponds to vertex Z). Example:



ΔABC is similar to ΔXYZ

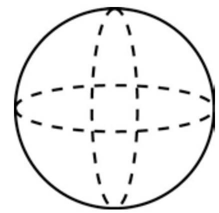
Sine (of an Angle) — A trigonometric ratio within a right triangle. The ratio is the length of the leg opposite the angle to the length of the hypotenuse of the triangle.

$$\text{sine of an angle} = \frac{\text{length of opposite leg}}{\text{length of hypotenuse}}$$

Skew Lines — Two lines that are not parallel and never intersect. Skew lines do not lie in the same plane.

Sphere

A three-dimensional figure or solid that has all points the same distance from the center. Informally, a perfectly round ball shape. Any cross-section of a sphere is circle. Example:



sphere

Straight Angle

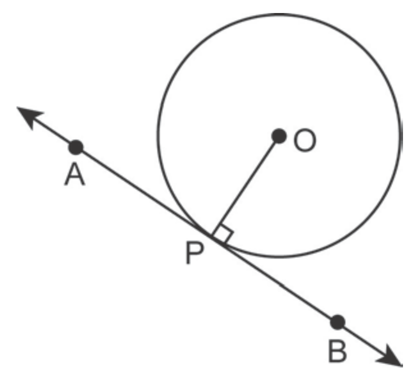
An angle that measures exactly 180°.

Surface Area

The total area of the surface of a three-dimensional figure. In a polyhedron, it is the sum of the areas of all the faces (i.e., two-dimensional surfaces).

Tangent (of a Circle)

A line, line segment, or ray that touches a circle at exactly one point. It is perpendicular to the radius at that point. Example:



$\overleftrightarrow{AB}$ is a tangent of circle O

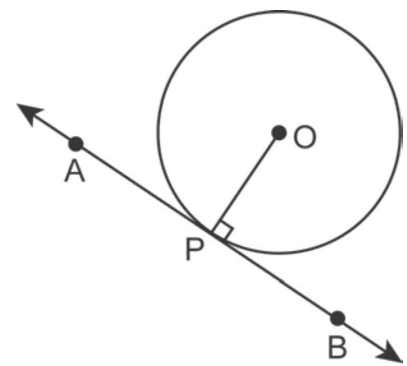
Tangent (of an Angle)

A trigonometric ratio within a right triangle. The ratio is the length of the leg opposite the angle to the length of the leg adjacent to the angle.

$$\text{tangent of an angle} = \frac{\text{length of opposite leg}}{\text{length of adjacent leg}}$$

Tangent (to a Circle)

A property of a line, line segment, or ray in that it touches a circle at exactly one point. It is perpendicular to the radius at that point. Example:

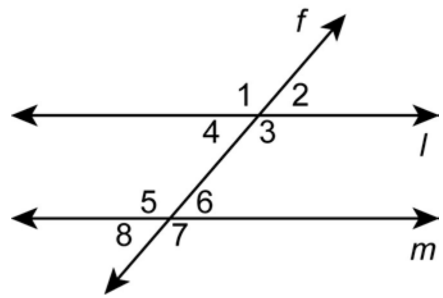


$\overleftrightarrow{AB}$ is tangent to circle O at point P

Three-Dimensional Figure

A figure that has three dimensions: length, width, and height. Three mutually perpendicular directions exist.

Transversal A line that crosses two or more lines intersecting each line at only one point to form eight or more angles. The lines that are crossed may or may not be parallel. Example:



line *f* is a transversal through parallel lines *l* and *m*

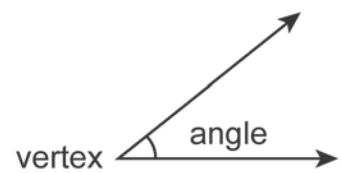
Trapezoid A quadrilateral with one pair of parallel sides, which are called the bases.

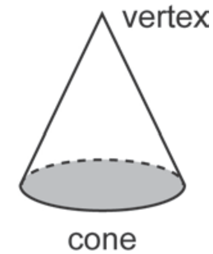
Triangle A three-sided polygon. The measures of its three interior angles add up to 180°. Triangles can be categorized by their angles, as acute, obtuse, right, or equiangular; or by their sides, as scalene, isosceles, or equilateral. A point where two of the three sides intersect is called a vertex. The symbol for a triangle is Δ (e.g., ΔABC is read "triangle ABC").

Trigonometric Ratio A ratio that compares the lengths of two sides of a right triangle and is relative to the measure of one of the angles in the triangle. The common ratios are sine, cosine, and tangent.

Two-Dimensional Figure A figure that has only two dimensions: length and width (no height). Two mutually perpendicular directions exist. Informally, it is "flat looking." The figure has area, but no volume.

Vertex A point where two or more rays meet, where two sides of a polygon meet, or where three (or more) edges of a polyhedron meet; the single point or apex of a cone. The plural is "vertices." Examples:





Volume The measure, in cubic units or units^3, of the amount of space contained by a three-dimensional figure or solid (i.e., the number of cubic units it takes to fill the figure).

Zero Angle An angle that measures exactly 0°.

Keystone Exams: Geometry

Assessment Anchors and Eligible Content
with Sample Questions and Glossary

April 2014