PA Core Standards
Standards for Mathematical Practice
Grade Level Emphasis*

The Pennsylvania Core Standards cannot be viewed and addressed in isolation, as each standard depends on or can lead into multiple standards across grades. Therefore, it is imperative that educators are familiar with both the standards that come before and those that follow a particular grade level. In addition, it is critical that the PA Core Standards of Mathematical Practice serve as the vehicle through which the PA Core Content Standards are taught.
This document is adapted from the Common Core State Standards for Mathematics.

The information below provides a grade progression of the Mathematical Practices to serve as guides for each on the eight practices. This is not intended to be a complete list but provide emphasized areas for an educator.

These eight practices can be clustered into the following categories as shown in the chart below and provide the foundation upon which the Transfer Goals within the curriculum framework were developed.

<table>
<tr>
<td rowspan="5">Habits of Mind of a Productive Mathematical Thinker
• Make sense of problems and persevere in solving them.
• Attend to precision.
•</td>
<td>Reasoning and Explaining
• Reason abstractly and quantitatively.
• Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr><td></td></tr>
<tr>
<td>Modeling and Using Tools
• Model with mathematics.
• Use appropriate tools strategically.</td>
</tr>
<tr><td></td></tr>
<tr>
<td>Seeing Structure and Generalizing
• Look for and make use of structure.
• Look for and express regularity in repeated reasoning.</td>
</tr>
</table>

In addition, a comprehensive definition of each practice is provided at the end of this document taken from the Common Core Standards for Mathematics.

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PA Core Standards
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Connecting the Standards for Mathematical Practice - Grade Progressions

Mathematical Practice	K	1	2
Students at this level will exhibit the following:			
Make sense of problems and persevere in solving them.	• Begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. • Explain to themselves the meaning of a problem and look for ways to solve it. • Use concrete objects or pictures to help them conceptualize and solve problems. • Check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.	• Realize that doing mathematics involves solving problems and discussing how they solved them. • Explain to themselves the meaning of a problem and look for ways to solve it. • Use concrete objects or pictures to help them conceptualize and solve problems. • Check their thinking by asking themselves, "Does this make sense?". • Willing to try other approaches.	• Realize that doing mathematics involves solving problems and discussing how they solved them. • Explain to themselves the meaning of a problem and look for ways to solve it. • Use concrete objects or pictures to help them conceptualize and solve problems. • Check their thinking by asking themselves, "Does this make sense?". • Make conjectures about the solution and plan out a problem solving approach.
Reason abstractly and quantitatively.	• Begin to recognize that a number represents a specific quantity. • Connect the quantity to written symbols. • Create a representation of a problem while attending to the meanings of the quantities (quantitative reasoning).	• Recognize that a number represents a specific quantity. • Connect the quantity to written symbols. • Create a representation of a problem while attending to the meanings of the quantities (quantitative reasoning).	• Recognize that a number represents a specific quantity. • Connect the quantity to written symbols. • Create a representation of a problem while attending to the meanings of the quantities (quantitative reasoning). • Begin to know and use different properties of operations and objects.

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PA Core Standards
Standards for Mathematical Practice
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Mathematical Practice	K	1	2
Students at this level will exhibit the following:			
Construct viable arguments and critique the reasoning of others.	• Construct arguments using concrete referents, such as objects, pictures, drawings, and actions. • Begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" • Explain their thinking to others and respond to others' thinking.	• Construct arguments using concrete referents, such as objects, pictures, drawings, and actions. • Practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Explain your thinking," and "Why is that true?" • Explain their own thinking, but listen to others' explanations. • Decide if the explanations make sense and ask questions.	• Construct arguments using concrete referents, such as objects, pictures, drawings, and actions. • Practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" "Explain your thinking," and "Why is that true?" • Explain their own thinking, but listen to others' explanations. • Decide if the explanations make sense and ask appropriate questions.
Model with mathematics.	• Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. • Connect the different representations and explain the connections. • Use all of these representations as needed.	• Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. • Connect the different representations and explain the connections. • Use all of these representations as needed.	• Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. • Connect the different representations and explain the connections. • Use all of these representations as needed.

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PA Core Standards
Standards for Mathematical Practice
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Mathematical Practice	K	1	2
Students at this level will exhibit the following:			
Use appropriate tools strategically.	• Begin to consider the available tools (including estimation) when solving a mathematical problem. • Decide when certain tools might be helpful. • Decide that it might be advantageous to use linking cubes to represent two quantities. • Compare the two representations side-by-side.	• Begin to consider the available tools (including estimation) when solving a mathematical problem. • Decide when certain tools might be helpful. • Decide it might be best to use colored chips to model an addition problem.	• Consider the available tools (including estimation) when solving a mathematical problem. • Decide when certain tools might be better suited. • Decide to solve a problem by drawing a picture rather than writing an equation.
Attend to precision.	• Develop their mathematical communication skills. • Use clear and precise language in their discussions with others and in their own reasoning.	• Develop their mathematical communication skills. • Use clear and precise language in their discussions with others and when they explain their own reasoning.	• Develop their mathematical communication skills. • Use clear and precise language in their discussions with others and when they explain their own reasoning.

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Mathematical Practice	K	1	2
Students at this level will exhibit the following:			
Look for and make use of structure.	• Begin to discern a pattern or structure. For instance, students recognize the pattern that exists in the teen numbers; every teen number is written with a 1 (representing one ten) and ends with the digit that is first stated. They also recognize that 3 + 2 = 5 and 2 + 3 = 5.	• Begin to discern a pattern or structure. For instance, if students recognize 12 + 3 = 15, then they also know 3 + 12 = 15 (Commutative property of addition). To add 4 + 6 + 4, the first two numbers can be added to make a ten, so 4 + 6 + 4 = 10 + 4 = 14.	• Look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).
Look for and express regularity in repeated reasoning.	• Notice repetitive actions in counting and computation, etc. For example, they may notice that the next number in a counting sequence is "one more". When counting by tens, the next number in the sequence is "ten more" (or one more group of ten). • Continually check their work by asking themselves, "Does this make sense?"	• Notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten" and multiples of "ten" they notice the pattern and gain a better understanding of place value. • Continually check their work by asking themselves, "Does this make sense?"	• Look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).

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Mathematical Practices	3	4	5
	Students at this level will exhibit the following:		
Make sense of problems and persevere in solving them.	• Know that doing mathematics involves solving problems and discussing how they solved them. • Explain to themselves the meaning of a problem and look for ways to solve it. • Use concrete objects or pictures to help them conceptualize and solve problems. • Check their thinking by asking themselves, "Does this make sense?" • Listen to the strategies of others and will try different approaches. • Use another method to check their answers.	• Know that doing mathematics involves solving problems and discussing how they solved them. • Explain to themselves the meaning of a problem and look for ways to solve it. • Use concrete objects or pictures to help them conceptualize and solve problems. • Check their thinking by asking themselves, "Does this make sense?" • Listen to the strategies of others and will try different approaches. • Use another method to check their answers.	• Solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. • Solve problems related to volume and measurement conversions. • Seek the meaning of a problem and look for efficient ways to represent and solve it. • Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
Reason abstractly and quantitatively.	• Recognize that a number represents a specific quantity. • Connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.	• Recognize that a number represents a specific quantity. • Connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. • Extend this understanding from whole numbers to their work with fractions and decimals. • Write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.	• Recognize that a number represents a specific quantity. • Connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. • Extend this understanding from whole numbers to their work with fractions and decimals. • Write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

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PA Core Standards
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Mathematical Practices	3	4	5
	Students at this level will exhibit the following:		
Construct viable arguments and critique the reasoning of others.	• Construct arguments using concrete referents, such as objects, pictures, and drawings. • Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" • Explain their thinking to others and respond to others' thinking.	• Construct arguments using concrete referents, such as objects, pictures, and drawings. • Explain their thinking and make connections between models and equations. • Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" • Explain their thinking to others and respond to others' thinking.	• Construct arguments using concrete referents, such as objects, pictures, and drawings. • Explain calculations based upon models and properties of operations and rules that generate patterns. • Demonstrate and explain the relationship between volume and multiplication. • Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" • Explain their thinking to others and respond to others' thinking.
Model with mathematics.	• Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. • Need opportunities to connect the different representations and explain the connections. • Use all of these representations as needed. • Evaluate their results in the context of the situation and reflect on whether the results make sense.	• Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. • Need opportunities to connect the different representations and explain the connections. • Use all of these representations as needed. • Evaluate their results in the context of the situation and reflect on whether the results make sense.	• Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. • Need opportunities to connect the different representations and explain the connections. Use all of these representations as needed. • Evaluate their results in the context of the situation and whether the results make sense. • Evaluate the utility of models to determine which models are most useful and efficient to solve problems.

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Mathematical Practices	3	4	5
	Students at this level will exhibit the following:		
Use appropriate tools strategically.	• Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. • Use graph paper to find all the possible rectangles that have a given perimeter. • Compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.	• Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. • Use graph paper or a number line to represent and compare decimals and protractors to measure angles. • Use other measurement tools to understand the relative size of units within a system. • Express measurements given in larger units in terms of smaller units.	• Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. • Use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. • Use graph paper to accurately create graphs. • Solve problems or make predictions from real world data.
Attend to precision.	• Develop their mathematical communication skills. • Use clear and precise language in their discussions with others and in their own reasoning. • Specify units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.	• Develop their mathematical communication skills. • Use clear and precise language in their discussions with others and in their own reasoning. • Specify units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.	• Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. • Use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. • Specify units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.

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Mathematical Practices	3	4	5
	Students at this level will exhibit the following:		
Look for and make use of structure.	• Look closely to discover a pattern or structure. • Use properties of operations as strategies to multiply and divide (commutative and distributive properties).	• Look closely to discover a pattern or structure. • Use properties of operations to explain calculations (partial products model). • Relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. • Generate number or shape patterns that follow a given rule.	• Look closely to discover a pattern or structure. • Use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. • Examine numerical patterns and relate them to a rule or a graphical representation.
Look for and express regularity in repeated reasoning.	• Notice repetitive actions in computation and look for more shortcut methods. • Use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. • Continually evaluate their work by asking themselves, "Does this make sense?"	• Notice repetitive actions in computation to make generalizations. • Use models to explain calculations and understand how algorithms work. • Use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.	• Use repeated reasoning to understand algorithms and make generalizations about patterns. • Connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers. • Perform all operations with decimals to hundredths. • Explore operations with fractions with visual models and begin to formulate generalizations.

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Mathematical Practice	6	7	8
	Students at this level will exhibit the following:		
Make sense of problems and persevere in solving them.	• Solve problems involving ratios and rates and discuss how they solved them. • Solve real world problems through the application of algebraic and geometric concepts. • Seek the meaning of a problem and look for efficient ways to represent and solve it. • Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"	• Solve problems involving ratios and rates and discuss how they solved them. • Solve real world problems through the application of algebraic and geometric concepts. • Seek the meaning of a problem and look for efficient ways to represent and solve it. • Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"	• Solve real world problems through the application of algebraic and geometric concepts. • Seek the meaning of a problem and look for efficient ways to represent and solve it. • Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
Reason abstractly and quantitatively.	• Represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. • Contextualize to understand the meaning of the number or variable as related to the problem. • Decontextualize to manipulate symbolic representations by applying properties of operations.	• Represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. • Contextualize to understand the meaning of the number or variable as related to the problem. • Decontextualize to manipulate symbolic representations by applying properties of operations.	• Represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. • Examine patterns in data and assess the degree of linearity of functions. • Contextualize to understand the meaning of the number or variable as related to the problem. • Decontextualize to manipulate symbolic representations by applying properties of operations.

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PA Core Standards
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Mathematical Practices	6	7	8
	Students at this level will exhibit the following:		
Construct viable arguments and critique the reasoning of others.	• Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). • Refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. • Pose questions like "How did you get that?", "Why is that true?" "Does that always work?" • Explain their thinking to others and respond to others' thinking.	• Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). • Refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. • Pose questions like "How did you get that?", "Why is that true?" "Does that always work?". • Explain their thinking to others and respond to others' thinking.	• Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). • Refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. • Pose questions like "How did you get that?", "Why is that true?" "Does that always work?" • Explain their thinking to others and respond to others' thinking.
Model with mathematics.	• Model problem situations symbolically, graphically, tabularly, and contextually. • Form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. • Begin to explore covariance and represent two quantities simultaneously. • Use number lines to compare numbers and represent inequalities. • Use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets. • Connect and explain the connections between the different representations. • Use all representations as appropriate to a problem context.	• Model problem situations symbolically, graphically, tabularly, and contextually. • Form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. • Explore covariance and represent two quantities simultaneously. • Use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. • Use experiments or simulations to generate data sets and create probability models. • Connect and explain the connections between the different representations. • Use all representations as appropriate to a problem context.	• Model problem situations symbolically, graphically, tabularly, and contextually. • Form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. • Solve systems of linear equations and compare properties of functions provided in different forms. • Use scatterplots to represent data and describe associations between variables. • Connect and explain the connections between the different representations. • Use all representations as appropriate to a problem context.

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Mathematical Practices	6	7	8
	Students at this level will exhibit the following:		
Use appropriate tools strategically.	• Consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. • Decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. • Use physical objects or applets to construct nets and calculate the surface area of three dimensional figures.	• Consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. • Decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. • Use physical objects or applets to generate probability data. • Use graphing calculators or spreadsheets to manage and represent data in different forms.	• Consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. • Translate a set of data given in tabular form to a graphical representation to compare it to another data set. • Draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal.
Attend to precision.	• Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. • Use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities.	• Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. • Define variables, specify units of measure, and label axes accurately. • Use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.	• Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. • Use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.

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Mathematical Practices	6	7	8
Students at this level will exhibit the following:			
Look for and make use of structure.	• Routinely seek patterns or structures to model and solve problems. • Recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties. • Apply properties to generate equivalent expressions (i.e. 6 + 2x = 3 (2 + x) by distributive property). • Solve equations (i.e. 2c + 3 = 15, 2c = 12 by subtraction property of equality, c=6 by division property of equality). • Compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume.	• Routinely seek patterns or structures to model and solve problems. • Recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. • Apply properties to generate equivalent expressions (i.e. 6 + 2x = 3 (2 + x) by distributive property). • Solve equations (i.e. 2c + 3 = 15, 2c = 12 by subtraction property of equality), c=6 by division property of equality). • Compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. • Examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.	• Routinely seek patterns or structures to model and solve problems. • Apply properties to generate equivalent expressions and solve equations. • Examine patterns in tables and graphs to generate equations and describe relationships. • Experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
Look for and express regularity in repeated reasoning.	• Use repeated reasoning to understand algorithms and make generalizations about patterns. • Solve and model problems. They may notice that a/b ÷ c/d = ad/bc and construct other examples and models that confirm their generalization. • Connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals. • Informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.	• Use repeated reasoning to understand algorithms and make generalizations about patterns. • Solve and model problems. They may notice that a/b ÷ c/d = ad/bc and construct other examples and models that confirm their generalization. • Extend their thinking to include complex fractions and rational numbers. • Formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. • Create, explain, evaluate, and modify probability models to describe simple and compound events.	• Use repeated reasoning to understand algorithms and make generalizations about patterns. • Use iterative processes to determine more precise rational approximations for irrational numbers. • Solve and model problems. They notice that the slope of a line and rate of change are the same value. • Flexibly make connections between covariance, rates, and representations showing the relationships between quantities.

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Mathematical Practices	High School
	Students at this level will exhibit the following:
1. Make sense of problems and persevere in solving them.	• Examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. • Analyze givens, constraints, relationships, and goals. • Make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. • Consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. • Monitor and evaluate their progress and change course if necessary. • Depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. • Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. • Check their answers to problems using different methods and continually ask themselves, "Does this make sense?" • Understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reason abstractly and quantitatively.	• Seek to make sense of quantities and their relationships in problem situations. • Abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. • Use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others.	• Understand and use stated assumptions, definitions, and previously established results in constructing arguments. • Make conjectures and build a logical progression of statements to explore the truth of their conjectures. • Analyze situations by breaking them into cases, and can recognize and use counterexamples. • Justify their conclusions, communicate them to others, and respond to the arguments of others. • Reason inductively about data, making plausible arguments that take into account the context from which the data arose. • Compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. • Determine domains, to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Mathematical Practices	High School
	Students at this level will exhibit the following:
4. Model with mathematics.	• Apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. • Use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. • Make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. • Identify important quantities in a practical situation and map their relationships using such tools as diagrams, two way tables, graphs, flowcharts and formulas. • Analyze relationships mathematically to draw conclusions. • Routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. Use appropriate tools strategically.	• Consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. • Be familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. • Analyze graphs of functions and solutions generated using a graphing calculator. • Detect possible errors by strategically using estimation and other mathematical knowledge. • Make mathematical models, knowing that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. • Identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. • Use technological tools to explore and deepen their understanding of concepts.
6. Attend to precision.	• Communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. • State the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. • Calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. • Examine claims and make explicit use of definitions.

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Mathematical Practices	High School
	Students at this level will exhibit the following:
7. Look for and make use of structure.	• Look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. • Recognize the significance of an existing line in a geometric figure and use the strategy of drawing an auxiliary line for solving problems. • Step back for an overview and shift perspective. • See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. • Use these patterns to create equivalent expressions, factor and solve equations, compose functions, and transform figures.
8. Look for and express regularity in repeated reasoning.	• Notice if calculations are repeated. Look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. • Derive formulas or make generalizations. • Maintain oversight of the process, while attending to the details. • Continually evaluate the reasonableness of their intermediate results.

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The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. **Make sense of problems and persevere in solving them.**
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.**
   Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.**

* This document is adapted from the Common Core State Standards for Mathematics.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.**
   Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.**
   Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at

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various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.**
   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.**
   Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression x 2 + 9x + 14, older students can see the 14 as 2×7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)$ 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.**
   Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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