COMMON CORE STATE STANDARDS FOR

Mathematics

Appendix A:

Designing High School Mathematics Courses Based on the Common Core State Standards

Overview

The Common Core State Standards (CCSS) for Mathematics are organized by grade level in Grades K–8. At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should learn in each category to be college and career ready, and to be prepared to study more advanced mathematics. As states consider how to implement the high school standards, an important consideration is how the high school CCSS might be organized into courses that provide a strong foundation for post-secondary success. To address this need, Achieve (in partnership with the Common Core writing team) has convened a group of experts, including state mathematics experts, teachers, mathematics faculty from two and four year institutions, mathematics teacher educators, and workforce representatives to develop Model Course Pathways in Mathematics based on the Common Core State Standards.

In considering this document, there are four things important to note:

1. The pathways and courses are models, not mandates. They illustrate possible approaches to organizing the content of the CCSS into coherent and rigorous courses that lead to college and career readiness. States and districts are not expected to adopt these courses as is; rather, they are encouraged to use these pathways and courses as a starting point for developing their own.

2. All college and career ready standards (those without a +) are found in each pathway. A few (+) standards are included to increase coherence but are not necessarily expected to be addressed on high stakes assessments.

3. The course descriptions delineate the mathematics standards to be covered in a course; they are not prescriptions for curriculum or pedagogy. Additional work will be needed to create coherent instructional programs that help students achieve these standards.

4. Units within each course are intended to suggest a possible grouping of the standards into coherent blocks; in this way, units may also be considered "critical areas" or "big ideas", and these terms are used interchangeably throughout the document. The ordering of the clusters within a unit follows the order of the standards document in most cases, not the order in which they might be taught. Attention to ordering content within a unit will be needed as instructional programs are developed.

5. While courses are given names for organizational purposes, states and districts are encouraged to carefully consider the content in each course and use names that they feel are most appropriate. Similarly, unit titles may be adjusted by states and districts.

While the focus of this document is on organizing the Standards for Mathematical Content into model pathways to college and career readiness, the content standards must also be connected to the Standards for Mathematical Practice to ensure that the skills needed for later success are developed. In particular, Modeling (defined by a * in the CCSS) is defined as both a *conceptual category* for high school mathematics and a *mathematical practice* and is an important avenue for motivating students to study mathematics, for building their understanding of mathematics, and for preparing them for future success. Development of the pathways into instructional programs will require careful attention to modeling and the mathematical practices. Assessments based on these pathways should reflect both the content and mathematical practices standards.

The Pathways

Four model course pathways are included:

1. An approach typically seen in the U.S. (Traditional) that consists of two algebra courses and a geometry course, with some data, probability and statistics included in each course;

2. An approach typically seen internationally (Integrated) that consists of a sequence of three courses, each of which includes number, algebra, geometry, probability and statistics;

3. A "compacted" version of the Traditional pathway where no content is omitted, in which students would complete the content of 7th grade, 8th grade, and the High School Algebra I course in grades 7 (Compacted 7th Grade) and 8 (8th Grade Algebra I), which will enable them to reach Calculus or other college level courses by their senior year. While the K-7 CCSS effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Accelerated 7th Grade course to make the 8th Grade Algebra I course more manageable;

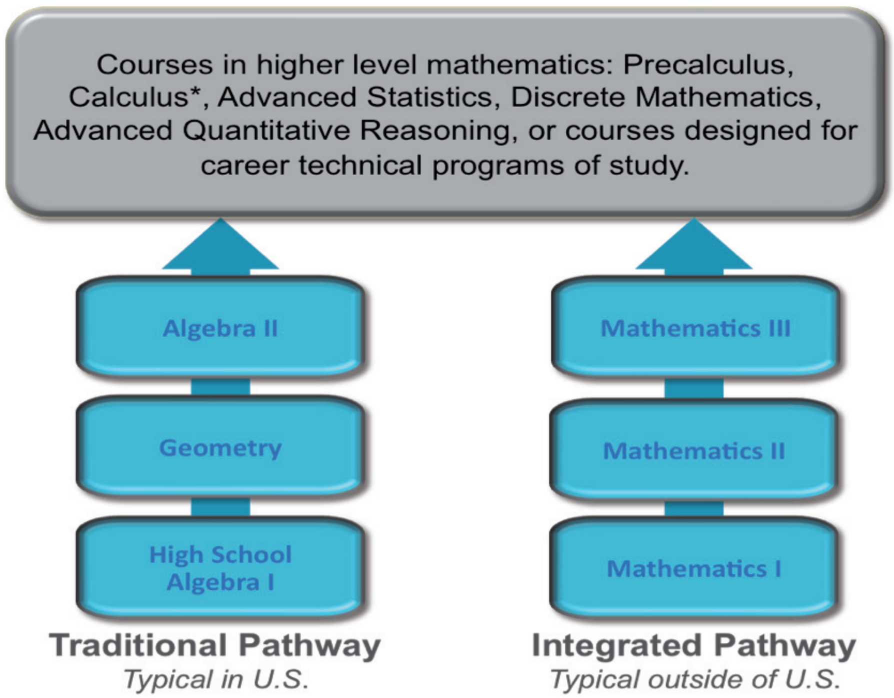
4. A "compacted" version of the Integrated pathway where no content is omitted, in which students would complete the content of 7th grade, 8th grade, and the Mathematics I course in grades 7 (Compacted 7th Grade) and 8 (8th Grade Mathematics I), which will enable them to reach Calculus or other college level courses by their senior year. While the K-7 CCSS effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Accelerated 7th Grade course to make the 8th Grade Mathematics I course more manageable;

5. Ultimately, all of these pathways are intended to significantly increase the coherence of high school mathematics.

The non-compacted, or regular, pathways assume mathematics in each year of high school and lead directly to preparedness for college and career readiness. In addition to the three years of study described in the Traditional and Integrated pathways, students should continue to take mathematics courses throughout their high school career to keep their mathematical understanding and skills fresh for use in training or course work after high school. A variety of courses should be available to students reflecting a range of possible interests; possible options are listed in the following chart. Based on a variety of inputs and factors, some students may decide at an early age that they want to take Calculus or other college level courses in high school. These students would need to begin the study of high school content in the middle school, which would lead to Precalculus or Advanced Statistics as a junior and Calculus, Advanced Statistics or other college level options as a senior.

Strategic use of technology is expected in all work. This may include employing technological tools to assist students in forming and testing conjectures, creating graphs and data displays and determining and assessing lines of fit for data. Geometric constructions may also be performed using geometric software as well as classical tools and technology may aid three-dimensional visualization. Testing with and without technological tools is recommended.

As has often occurred in schools and districts across the states, greater resources have been allocated to accelerated pathways, such as more experienced teachers and newer materials. The Achieve Pathways Group members strongly believe that each pathway should get the same attention to quality and resources including class sizes, teacher assignments, professional development, and materials. Indeed, these and other pathways should be avenues for students to pursue interests and aspirations. The following flow chart shows how the courses in the two regular pathways are sequenced (the * in the chart on the following page means that Calculus follows Precalculus and is a fifth course, in most cases). More information about the compacted pathways can be found later in this appendix.



Some teachers and schools are effectively getting students to be college and career ready. We can look to these teachers and schools to see what kinds of courses are getting results, and to compare pathways courses to the mathematics taught in effective classrooms.

A study done by ACT and The Education Trust gives evidence to support these pathways. The study looked at high-poverty schools where a high percentage of students were reaching and exceeding ACT's college-readiness benchmarks. From these schools, the most effective teachers described their courses and opened up their classrooms for observation. The commonality of mathematics topics in their courses gives a picture of what it takes to get students to succeed, and also provides a grounding for the pathways. (There were other commonalities. For more detailed information about this study, search for the report On Course for Success at www.act.org.)[1]

Implementation Considerations:

As states, districts and schools take on the work of implementing the Common Core State Standards, the Model Course Pathways in Mathematics can be a useful foundation for discussing how best to organize the high school standards into courses. The Pathways have been designed to be modular in nature, where the modules or critical areas (units) are identical in nearly every manner between the two pathways, but are arranged in different orders to accommodate different organizational offerings. Assessment developers may consider the creation of assessment modules in a similar fashion. Curriculum designers may create alternative model pathways with altogether different organizations of the standards. Some of this work is already underway. In short, this document is intended to contribute to the conversations around assessment and curriculum design, rather than end them. Effectively implementing these standards will require a long-term commitment to understanding what best supports student learning and attainment of college and career readiness skills by the end of high school, as well as regular revision of pathways as student learning data becomes available.

Supporting Students

One of the hallmarks of the Common Core State Standards for Mathematics is the specification of content that all students must study in order to be college and career ready. This "college and career ready line" is a minimum for all students. However, this does not mean that all students should progress uniformly to that goal. Some students progress

[1] The study provides evidence that the pathways' High School Algebra I, Geometry, Algebra II sequence is a reasonable and rigorous option for preparing students for college and career. Topics aligned almost completely between the CCSS topics and topics taught in the study classrooms. The starting point for the pathways' High School Algebra I course is slightly beyond the starting point for the study Algebra I courses due to the existence of many typical Algebra I topics in the 8th grade CCSS, therefore some of the study Algebra II topics are a part of the pathways' High School Algebra I course, specifically, using the quadratic formula; a bit more with exponential functions including comparing and contrasting linear and exponential growth; and the inclusion of the spread of data sets. The pathways' Geometry course is very similar to what was done in the study Geometry courses, with the addition of the laws of sines and cosines and the work with conditional probability, plus applications involving completing the square because that topic was part of the pathways' High School Algebra I course. The pathways' Algebra II course then matches well with what was done in the study Algebra II courses and continues a bit into what was done in the study Precalculus classrooms, including inverse functions, the behavior of logarithmic and trigonometric functions, and in statistics with the normal distribution, margin of error, and the differences among sample surveys, experiments, and observational studies. All in all, the topics and the order of topics is very comparable between the pathways' High School Algebra I, Geometry, Algebra II sequence and the sequence found in the study courses.

more slowly than others. These students will require additional support, and the following strategies, consistent with Response to Intervention practices, may be helpful:

- Creating a school-wide community of support for students;
- Providing students a “math support” class during the school day;
- After-school tutoring;
- Extended class time (or blocking of classes) in mathematics; and
- Additional instruction during the summer.

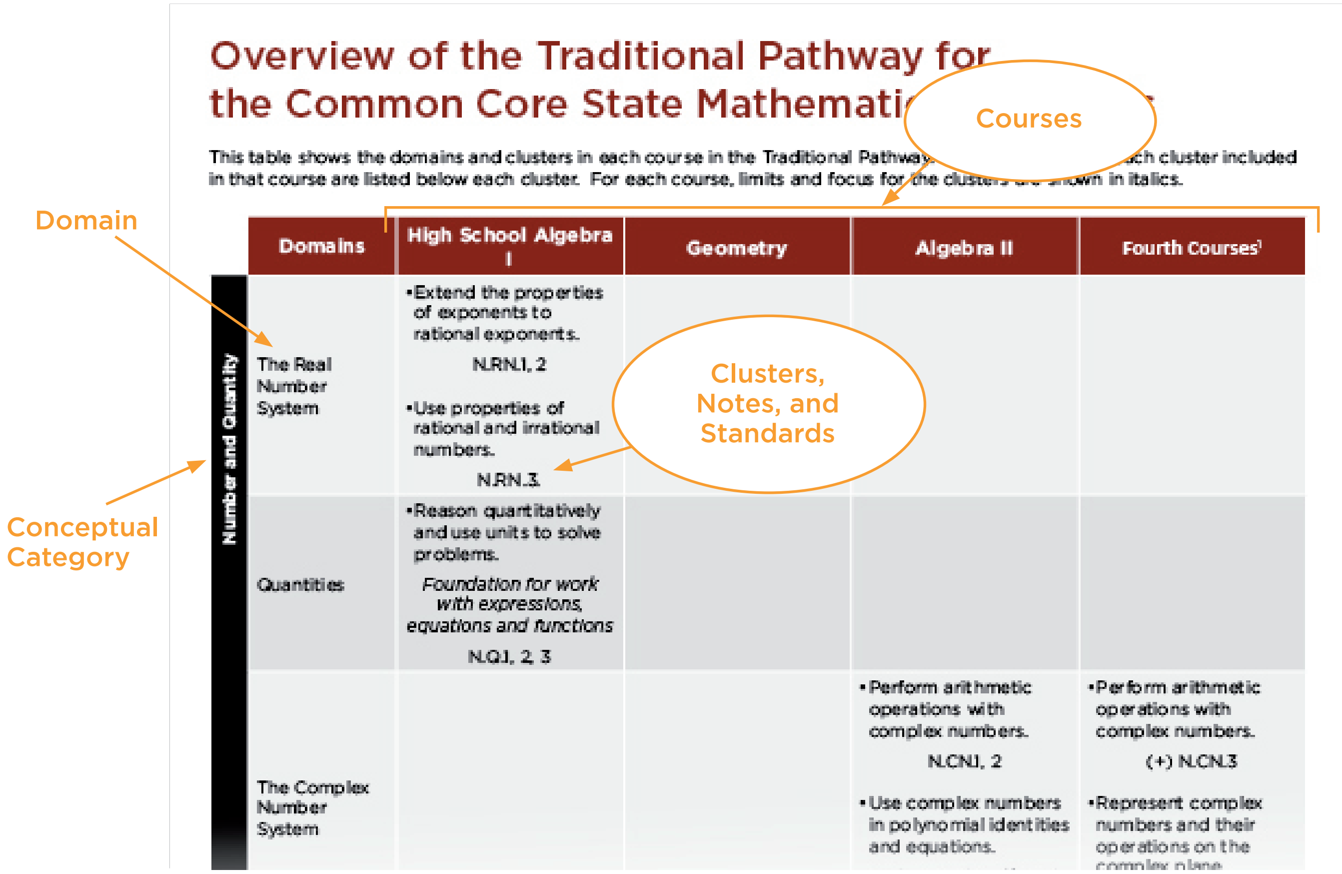
Watered-down courses which leave students uninspired to learn, unable to catch up to their peers and unready for success in postsecondary courses or for entry into many skilled professions upon graduation from high school are neither necessary nor desirable. The results of not providing students the necessary supports they need to succeed in high school are well-documented. Too often, after graduation, such students attempt to continue their education at 2- or 4-year postsecondary institutions only to find they must take remedial courses, spending time and money mastering high school level skills that they should have already acquired. This, in turn, has been documented to indicate a greater chance of these students not meeting their postsecondary goals, whether a certificate program, two- or four-year degree. As a result, in the workplace, many career pathways and advancement may be denied to them. To ensure students graduate fully prepared, those who enter high school underprepared for high school mathematics courses must receive the support they need to get back on course and graduate ready for life after high school.

Furthermore, research shows that allowing low-achieving students to take low-level courses is not a recipe for academic success (Kifer, 1993). The research strongly suggests that the goal for districts should not be to stretch the high school mathematics standards over all four years. Rather, the goal should be to provide support so that all students can reach the college and career ready line by the end of the eleventh grade, ending their high school career with one of several high-quality mathematical courses that allows students the opportunity to deepen their understanding of the college- and career-ready standards.

With the Common Core State Standards Initiative comes an unprecedented ability for schools, districts, and states to collaborate. While this is certainly the case with respect to assessments and professional development programs, it is also true for strategies to support struggling and accelerated students. The Model Course Pathways in Mathematics are intended to launch the conversation, and give encouragement to all educators to collaborate for the benefit of our states’ children.

How to Read the Pathways:

Each pathway consists of two parts. The first is a chart that shows an overview of the pathway. Organized by course and by conceptual category (algebra, functions, geometry, etc…), these charts show which clusters and standards appear in which course (see page 5 of the CCSS for definitions of clusters and standards). For example, in the chart below, the three standards (N.Q.1, 2, 3) associated with the cluster "Reason quantitatively and use units to solve problems," are found in Course 1. This cluster is found under the domain "Quantities" in the "Number and Quantity" conceptual category. All high school standards in the CCSS are located in at least one of the courses in this chart.

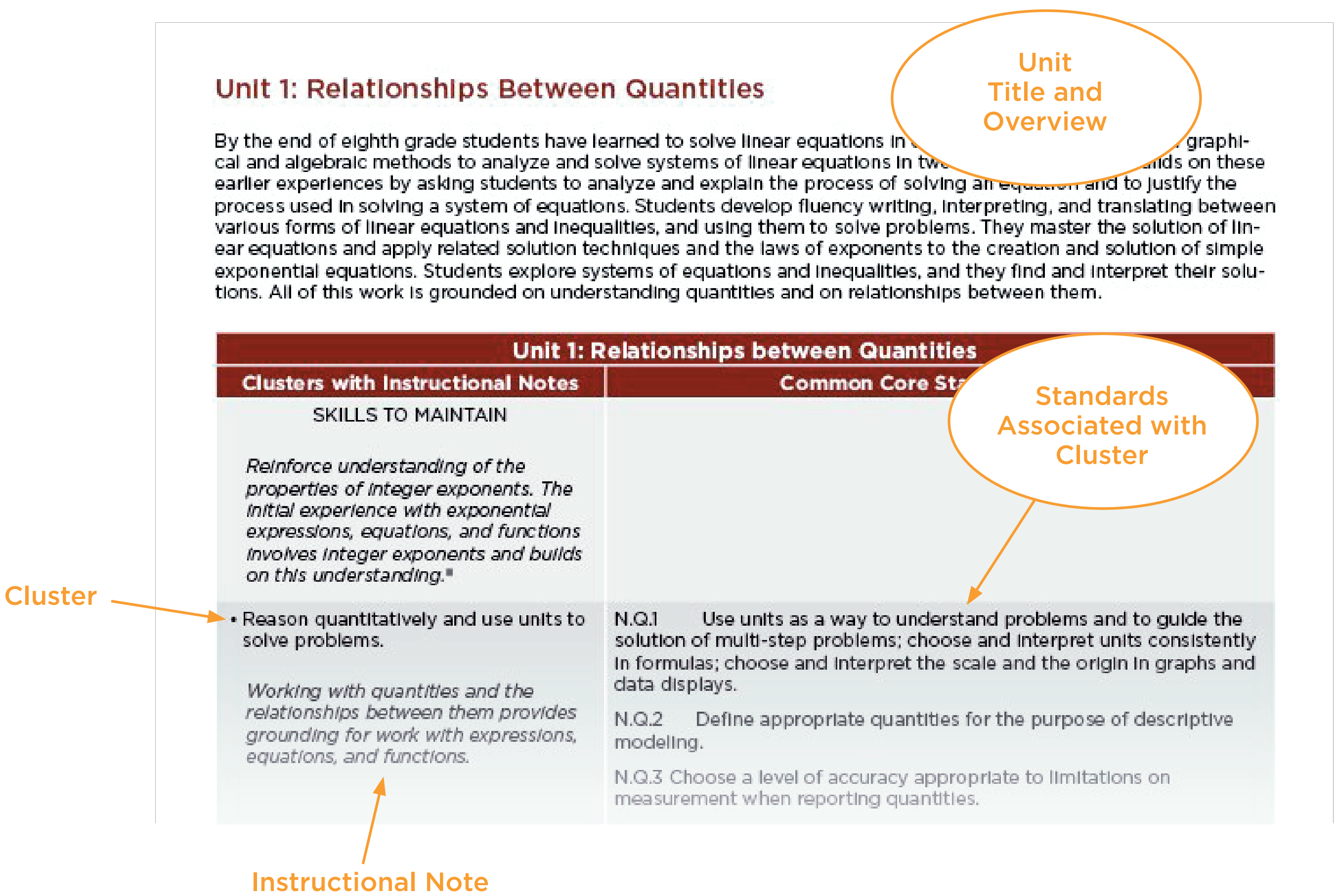


Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses[1]
The Real Number System	•Extend the properties of exponents to rational exponents. N.RN.1, 2 •Use properties of rational and irrational numbers. N.RN.3			
Quantities	•Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions* N.Q.1, 2, 3			
The Complex Number System			•Perform arithmetic operations with complex numbers. N.CN.1, 2 •Use complex numbers in polynomial identities and equations.	•Perform arithmetic operations with complex numbers. (+) N.CN.3 •Represent complex numbers and their operations on the complex plane

The second part of the pathways shows the clusters and standards as they appear in the courses. Each course contains the following components:

- An introduction to the course and a list of the units in the course
- Unit titles and unit overviews (see below)
- Units that show the cluster titles, associated standards, and instructional notes (below)

It is important to note that the units (or critical areas) are intended to convey coherent groupings of content. The clusters and standards within units are ordered as they are in the Common Core State Standards, and are not intended to convey an instructional order. Considerations regarding constraints, extensions, and connections are found in the instructional notes. The instructional notes are a critical attribute of the courses and should not be overlooked. For example, one will see that standards such as A.CED.1 and A.CED.2 are repeated in multiple courses, yet their emphases change from one course to the next. These changes are seen only in the instructional notes, making the notes an indispensable component of the pathways.



Unit 1: Relationships Between Quantities

By the end of eighth grade students have learned to solve linear equations in … graphi-cal and algebraic methods to analyze and solve systems of linear equations in tw… ds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Unit 1: Relationships between Quantities	
Clusters with Instructional Notes	**Common Core Sta…**
SKILLS TO MAINTAIN *Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.**	
• Reason quantitatively and use units to solve problems. *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Overview of the Traditional Pathway for the Common Core State Mathematics Standards

This table shows the domains and clusters in each course in the Traditional Pathway. The standards from each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics.

	Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses*
Number and Quantity	The Real Number System	• Extend the properties of exponents to rational exponents. N.RN.1, 2 • Use properties of rational and irrational numbers. N.RN.3			
	Quantities	• Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions* N.Q.1, 2, 3			
	The Complex Number System			• Perform arithmetic operations with complex numbers. N.CN.1, 2 • Use complex numbers in polynomial identities and equations. *Polynomials with real coefficients* N.CN.7, (+) 8, (+) 9	• Perform arithmetic operations with complex numbers. (+) N.CN.3 • Represent complex numbers and their operations on the complex plane. (+) N.CN.4, 5, 6
	Vector Quantities and Matrices				• Represent and model with vector quantities. (+) N.VM.1, 2, 3 • Perform operations on vectors. (+) N.VM.4a, 4b, 4c, 5a, 5b • Perform operations on matrices and use matrices in applications. (+) N.VM.6, 7, 8, 9, 10, 11, 12

*The (+) standards in this column are those in the Common Core State Standards that are not included in any of the Traditional Pathway courses. They would be used in additional courses developed to follow Algebra II.

	Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses
Algebra	Seeing Structure in Expressions	• Interpret the structure of expressions. *Linear, exponential, quadratic* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. *Quadratic and exponential* A.SSE.3a, 3b, 3c		• Interpret the structure of expressions. *Polynomial and rational* *A.SSE.1a, 1b, 2* • Write expressions in equivalent forms to solve problems. A.SSE.4	
	Arithmetic with Polynomials and Rational Expressions	• Perform arithmetic operations on polynomials. *Linear and quadratic* A.APR.1		• Perform arithmetic operations on polynomials. *Beyond quadratic* A.APR.1 • Understand the relationship between zeros and factors of polynomials. A.APR.2, 3 • Use polynomial identities to solve problems. A.APR.4, (+) 5 • Rewrite rational expressions. *Linear and quadratic denominators* A.APR.6, (+) 7	
	Creating Equations	• Create equations that describe numbers or relationships. *Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only* A.CED.1, 2, 3, 4		• Create equations that describe numbers or relationships. *Equations using all available types of expressions, including simple root functions* A.CED.1, 2, 3, 4	

	Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses
Algebra	Reasoning with Equations and Inequalities	• Understand solving equations as a process of reasoning and explain the reasoning. *Master linear; learn as general principle* A.REI.1 • Solve equations and inequalities in one variable. *Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions* A.REI.3, 4a, 4b • Solve systems of equations. *Linear-linear and linear-quadratic* A.REI.5, 6, 7 • Represent and solve equations and inequalities graphically. *Linear and exponential; learn as general principle* A.REI.10, 11, 12		• Understand solving equations as a process of reasoning and explain the reasoning. *Simple radical and rational* A.REI.2 • Represent and solve equations and inequalities graphically. *Combine polynomial, rational, radical, absolute value, and exponential functions* A.REI.11	• Solve systems of equations. (+) A.REI.8, 9
Functions	Interpreting Functions	• Understand the concept of a function and use function notation. *Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences* F.IF.1, 2, 3 • Interpret functions that arise in applications in terms of a context. *Linear, exponential, and quadratic* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear, exponential, quadratic, absolute value, step, piecewise-defined* F.IF.7a, 7b, 7e, 8a, 8b, 9		• Interpret functions that arise in applications in terms of a context. *Emphasize selection of appropriate models* F.IF.4, 5, 6 • Analyze functions using different representations. *Focus on using key features to guide selection of appropriate type of model function* F.IF.7b, 7c, 7e, 8, 9	• Analyze functions using different representations. *Logarithmic and trigonometric functions* (+) F.IF.7d

	Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses
Functions	Building Functions	• Build a function that models a relationship between two quantities. *For F.BF.1, 2, linear, exponential, and quadratic* F.BF.1a, 1b, 2 • Build new functions from existing functions. *Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only* F.BF.3, 4a		• Build a function that models a relationship between two quantities. *Include all types of functions studied* F.BF.1b • Build new functions from existing functions. *Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types* F.BF.3, 4a	• Build a function that models a relationship between two quantities. (+) F.BF.1c • Build new functions from existing functions. (+) F.BF.4b, 4c, 4d, 5
	Linear, Quadratic, and Exponential Models	• Construct and compare linear, quadratic, and exponential models and solve problems. F.LE.1a, 1b, 1c, 2, 3 • Interpret expressions for functions in terms of the situation they model. *Linear and exponential of form $f(x)=b^x+k$* F.LE.5		• Construct and compare linear, quadratic, and exponential models and solve problems. *Logarithms as solutions for exponentials* F.LE.4	
	Trigonometric Functions			• Extend the domain of trigonometric functions using the unit circle. F.TF.1, 2 • Model periodic phenomena with trigonometric functions. F.TF.5 • Prove and apply trigonometric identities. F.TF.8	• Extend the domain of trigonometric functions using the unit circle. (+) F.TF.3, 4 • Model periodic phenomena with trigonometric functions. (+) F.TF. 6, 7 • Prove and apply trigonometric identities. (+) F.TF. 9

	Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses
Geometry	Congruence		• Experiment with transformations in the plane. G.CO.1, 2, 3, 4, 5 • Understand congruence in terms of rigid motions. *Build on rigid motions as a familiar starting point for development of concept of geometric proof* G.CO.6, 7, 8 • Prove geometric theorems. *Focus on validity of underlying reasoning while using variety of ways of writing proofs* G.CO.9, 10, 11 • Make geometric constructions. *Formalize and explain processes* G.CO.12, 13		
	Similarity, Right Triangles, and Trigonometry		• Understand similarity in terms of similarity transformations. G.SRT.1a, 1b, 2, 3 • Prove theorems involving similarity. G.SRT.4, 5 • Define trigonometric ratios and solve problems involving right triangles. G.SRT.6, 7, 8 • Apply trigonometry to general triangles. G.SRT.9. 10, 11		

	Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses
Geometry	Circles		• Understand and apply theorems about circles. G.C.1, 2, 3, (+) 4 • Find arc lengths and areas of sectors of circles. *Radian introduced only as unit of measure* G.C.5		
	Expressing Geometric Properties with Equations		• Translate between the geometric description and the equation for a conic section. G.GPE.1, 2 • Use coordinates to prove simple geometric theorems algebraically. *Include distance formula; relate to Pythagorean theorem* G.GPE. 4, 5, 6, 7		• Translate between the geometric description and the equation for a conic section. (+) G.GPE.3
	Geometric Measurement and Dimension		• Explain volume formulas and use them to solve problems. G.GMD.1, 3 • Visualize the relation between two-dimensional and three-dimensional objects. G.GMD.4		• Explain volume formulas and use them to solve problems. (+) G.GMD.2
	Modeling with Geometry		• Apply geometric concepts in modeling situations. G.MG.1, 2, 3		
Statistics and Probability	Interpreting Categorical and Quantitative Data	• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.1, 2, 3 • Summarize, represent, and interpret data on two categorical and quantitative variables. *Linear focus, discuss general principle* S.ID.5, 6a, 6b, 6c • Interpret linear models S.ID.7, 8, 9		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.4	

Statistics and Probability

Domains	High School Algebra I	Geometry	Algebra II	Fourth Courses
Making Inferences and Justifying Conclusions			• Understand and evaluate random processes underlying statistical experiments. S.IC.1, 2 • Make inferences and justify conclusions from sample surveys, experiments and observational studies. S.IC.3, 4, 5, 6	
Conditional Probability and the Rules of Probability		• Understand independence and conditional probability and use them to interpret data. *Link to data from simulations or experiments* S.CP.1, 2, 3, 4, 5 • Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.6, 7, (+) 8, (+) 9		
Using Probability to Make Decisions		• Use probability to evaluate outcomes of decisions. *Introductory; apply counting rules* (+) S.MD.6, 7	• Use probability to evaluate outcomes of decisions. *Include more complex situations* (+) S.MD.6, 7	• Calculate expected values and use them to solve problems. (+) S.MD.1, 2, 3, 4 • Use probability to evaluate outcomes of decisions.. (+) S.MD. 5a, 5b

Traditional Pathway: High School Algebra I

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Relationships Between Quantities and Reasoning with Equations</td><td>• Reason quantitatively and use units to solve problems.
• Interpret the structure of expressions.
• Create equations that describe numbers or relationships.
• Understand solving equations as a process of reasoning and explain the reasoning.
• Solve equations and inequalities in one variable.</td><td rowspan="5">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Linear and Exponential Relationships</td><td>• Extend the properties of exponents to rational exponents.
• Solve systems of equations.
• Represent and solve equations and inequalities graphically.
• Understand the concept of a function and use function notation.
• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.
• Interpret expressions for functions in terms of the situation they model.</td></tr>
<tr><td>Unit 3
Descriptive Statistics</td><td>• Summarize, represent, and interpret data on a single count or measurement variable.
• Summarize, represent, and interpret data on two categorical and quantitative variables.
• Interpret linear models.</td></tr>
<tr><td>Unit 4
Expressions and Equations</td><td>• Interpret the structure of expressions.
• Write expressions in equivalent forms to solve problems.
• Perform arithmetic operations on polynomials.
• Create equations that describe numbers or relationships.
• Solve equations and inequalities in one variable.
• Solve systems of equations.</td></tr>
<tr><td>Unit 5
Quadratic Functions and Modeling</td><td>• Use properties of rational and irrational numbers.
• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Relationships Between Quantities and Reasoning with Equations

By the end of eighth grade students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All of this work is grounded on understanding quantities and on relationships between them.

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	**Common Core State Standards**
SKILLS TO MAINTAIN *Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.**	
• Reason quantitatively and use units to solve problems. *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
• Interpret the structure of expressions. *Limit to linear expressions and to exponential expressions with integer exponents.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
• Create equations that describe numbers or relationships. *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variable of interest.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance R.*

*Instructional suggestions will be found in italics in this column throughout the document.

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Understand solving equations as a process of reasoning and explain the reasoning. *Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II.*	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
• Solve equations and inequalities in one variable. *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as* $5^x=125$ or $2^x=1/16$.	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Extend the properties of exponents to rational exponents. *In implementing the standards in curriculum, these standards should occur before discussing exponential functions with continuous domains.*	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
• Solve systems of equations. *Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5 when it is taught in Geometry, which requires students to prove the slope criteria for parallel lines.*	A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
• Represent and solve equations and inequalities graphically. *For A.REI.10, focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.*	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Understand the concept of a function and use function notation. *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of functions at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.* *Draw examples from linear and exponential functions. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.*	F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*
• Interpret functions that arise in applications in terms of a context. *For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and exponential functions whose domain is a subset of the integers. Unit 5 in this course and the Algebra II course address other types of functions.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7a, 7e, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100^2$*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Build a function that models a relationship between two quantities. *Limit to F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
• Build new functions from existing functions. *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.* *While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
• Construct and compare linear, quadratic, and exponential models and solve problems. *For F.LE.3, limit to comparisons between linear and exponential models. In constructing linear functions in F.LE.2, draw on and consolidate previous work in Grade 8 on finding equations for lines and linear functions (8.EE.6, 8.F.4).*	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
• Interpret expressions for functions in terms of the situation they model. *Limit exponential functions to those of the form* $f(x) = b^x + k$.	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Unit 3: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatter-plots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 3: Descriptive Statistics	
Clusters with Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.*	S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
• Summarize, represent, and interpret data on two categorical and quantitative variables. *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.* *S.ID.6b should be focused on linear models, but may be used to preview quadratic functions in Unit 5 of this course.*	S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
• Interpret linear models. *Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.*	S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.9 Distinguish between correlation and causation.

Unit 4: Expressions and Equations

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret the structure of expressions. *Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Unit 1 to rational exponents focusing on those that represent square or cube roots.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
• Write expressions in equivalent forms to solve problems. *It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.*	A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
• Perform arithmetic operations on polynomials. *Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.*	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
• Create equations that describe numbers or relationships. *Extend work on linear and exponential equations in Unit 1 to quadratic equations. Extend A.CED.4 to formulas involving squared variables.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*
• Solve equations and inequalities in one variable. *Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.*	A.REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Solve systems of equations. *Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2+y^2=1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2+4^2=5^2$.*	A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.*

Unit 5: Quadratic Functions and Modeling

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Use properties of rational and irrational numbers. *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*	N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
• Interpret functions that arise in applications in terms of a context. *Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.* *Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.* F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Build a function that models a relationship between two quantities. *Focus on situations that exhibit a quadratic relationship.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
• Build new functions from existing functions. *For F.BF.3, focus on quadratic functions, and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x>0$.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2 x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*
• Construct and compare linear, quadratic, and exponential models and solve problems. *Compare linear and exponential growth to quadratic growth.*	F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Traditional Pathway: Geometry

TheThe fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into six units are as follows.

Critical Area 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Critical Area 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4: Building on their work with the Pythagorean theorem in 8^{th} grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Critical Area 5: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.

Critical Area 6: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Congruence, Proof, and Constructions</td><td>Experiment with transformations in the plane.
Understand congruence in terms of rigid motions.
Prove geometric theorems.
Make geometric constructions.</td><td rowspan="6">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Similarity, Proof, and Trigonometry</td><td>Understand similarity in terms of similarity transformations.
Prove theorems involving similarity.
Define trigonometric ratios and solve problems involving right triangles.
Apply geometric concepts in modeling situations.
Apply trigonometry to general triangles.</td></tr>
<tr><td>Unit 3
Extending to Three Dimensions</td><td>Explain volume formulas and use them to solve problems.
Visualize the relation between two-dimensional and three-dimensional objects.
Apply geometric concepts in modeling situations.</td></tr>
<tr><td>Unit 4
Connecting Algebra and Geometry through Coordinates</td><td>Use coordinates to prove simple geometric theorems algebraically.
Translate between the geometric description and the equation for a conic section.</td></tr>
<tr><td>Unit 5
Circles With and Without Coordinates</td><td>Understand and apply theorems about circles.
Find arc lengths and areas of sectors of circles.
Translate between the geometric description and the equation for a conic section.
Use coordinates to prove simple geometric theorem algebraically.
Apply geometric concepts in modeling situations.</td></tr>
<tr><td>Unit 6
Applications of Probability</td><td>Understand independence and conditional probability and use them to interpret data.
Use the rules of probability to compute probabilities of compound events in a uniform probability model.
Use probability to evaluate outcomes of decisions.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 1: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	**Common Core State Standards**
• Experiment with transformations in the plane. *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.*	G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
• Understand congruence in terms of rigid motions. *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.*	G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
• Prove geometric theorems. *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 5.*	G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Unit 1: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	**Common Core State Standards**
• Make geometric constructions. *Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects.* *Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.*	G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 2: Similarity, Proof, and Trigonometry

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.

Unit 2: Similarity, Proof, and Trigonometry	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand similarity in terms of similarity transformations.	G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
• Prove theorems involving similarity.	G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*. G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
• Define trigonometric ratios and solve problems involving right triangles.	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★
• Apply geometric concepts in modeling situations. *Focus on situations well modeled by trigonometric ratios for acute angles.*	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
• Apply trigonometry to general triangles. *With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.*	G.SRT.9 (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Unit 3: Extending to Three Dimensions

Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Unit 3: Extending to Three Dimensions	
Clusters and Instructional Notes	**Common Core State Standards**
• Explain volume formulas and use them to solve problems. *Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.*	G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★
• Visualize the relation between two-dimensional and three-dimensional objects.	G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
• Apply geometric concepts in modeling situations. *Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.*	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

Unit 4: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Unit 4: Connecting Algebra and Geometry Through Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Use coordinates to prove simple geometric theorems algebraically. *This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.* *Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in High School Algebra I involving systems of equations having no solution or infinitely many solutions.* *G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★
• Translate between the geometric description and the equation for a conic section. *The directrix should be parallel to a coordinate axis.*	G.GPE.2 Derive the equation of a parabola given a focus and directrix.

Unit 5: Circles With and Without Coordinates

In this unit, students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or parabolas and between two circles.

Unit 5: Circles With and Without Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand and apply theorems about circles.	G.C.1 Prove that all circles are similar. G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.
• Find arc lengths and areas of sectors of circles. *Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.*	G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
• Translate between the geometric description and the equation for a conic section.	G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
• Use coordinates to prove simple geometric theorems algebraically. *Include simple proofs involving circles.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
• Apply geometric concepts in modeling situations. *Focus on situations in which the analysis of circles is required.*	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

Unit 6: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Unit 6: Applications of Probability	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand independence and conditional probability and use them to interpret data. *Build on work with two-way tables from Algebra I Unit 3 (S.ID.5) to develop understanding of conditional probability and independence.*	S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.3 Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
• Use the rules of probability to compute probabilities of compound events in a uniform probability model.	S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model. S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
• Use probability to evaluate outcomes of decisions. *This unit sets the stage for work in Algebra II, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.*	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Traditional Pathway: Algebra II

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions.[2] Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas for this course, organized into four units, are as follows:

Critical Area 1: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 2: Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Critical Area 3: In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Critical Area 4: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

[2]In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Polynomial, Rational, and Radical Relationships</td><td>• Perform arithmetic operations with complex numbers.
• Use complex numbers in polynomial identities and equations.
• Interpret the structure of expressions.
• Write expressions in equivalent forms to solve problems.
• Perform arithmetic operations on polynomials.
• Understand the relationship between zeros and factors of polynomials.
• Use polynomial identities to solve problems.
• Rewrite rational expressions.
• Understand solving equations as a process of reasoning and explain the reasoning.
• Represent and solve equations and inequalities graphically.
• Analyze functions using different representations.</td><td rowspan="4">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Trigonometric Functions</td><td>• Extend the domain of trigonometric functions using the unit circle.
• Model periodic phenomena with trigonometric function.
• Prove and apply trigonometric identites.</td></tr>
<tr><td>Unit 3
Modeling with Functions</td><td>• Create equations that describe numbers or relationships.
• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.</td></tr>
<tr><td>Unit 4
Inferences and Conclusions from Data</td><td>• Summarize, represent, and interpret data on single count or measurement variable.
• Understand and evaluate random processes underlying statistical experiments.
• Make inferences and justify conclusions from sample surveys, experiments and observational studies.
• Use probability to evaluate outcomes of decisions.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Polynomial, Rational, and Radical Relationships

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Unit 1: Polynomial, Rational, and Radical Relationships	
Clusters and Instructional Notes	**Common Core State Standards**
• Perform arithmetic operations with complex numbers.	N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
• Use complex numbers in polynomial identities and equations. *Limit to polynomials with real coefficients.*	N.CN.7 Solve quadratic equations with real coefficients that have complex solutions. N.CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
• Interpret the structure of expressions. *Extend to polynomial and rational expressions.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
• Write expressions in equivalent forms to solve problems. *Consider extending A.SSE.4 to infinite geometric series in curricular implementations of this course description.*	A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★
• Perform arithmetic operations on polynomials. *Extend beyond the quadratic polynomials found in Algebra I.*	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
• Understand the relationship between zeros and factors of polynomials.	A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only *if* $(x - a)$ is a factor of p(x). A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Unit 1: Polynomial, Rational, and Radical Relationships	
Clusters and Instructional Notes	**Common Core State Standards**
• Use polynomial identities to solve problems. *This cluster has many possibilities for optional enrichment, such as relating the example in A.APR.4 to the solution of the system $u^2+v^2=1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x+y)^{n+1} = (x+y)(x+y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.*	A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
• Rewrite rational expressions *The limitations on rational functions apply to the rational expressions in A.APR.6. A.APR.7 requires the general division algorithm for polynomials.*	A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
• Understand solving equations as a process of reasoning and explain the reasoning. *Extend to simple rational and radical equations.*	A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
• Represent and solve equations and inequalities graphically. *Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.*	A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
• Analyze functions using different representations. *Relate F.IF.7c to the relationship between zeros of quadratic functions and their factored forms*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Unit 2: Trigonometric Functions

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Unit 2: Trigonometric Functions	
Clusters and Instructional Notes	**Common Core State Standards**
• Extend the domain of trigonometric functions using the unit circle.	F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
• Model periodic phenomena with trigonometric functions.	F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
• Prove and apply trigonometric identities. *An Algebra II course with an additional focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could be limited to acute angles in Algebra II.*	F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle.

Unit 3: Modeling with Functions

In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. *The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.*

Unit 3: Modeling with Functions	
Clusters and Instructional Notes	**Common Core State Standards**
• Create equations that describe numbers or relationships. *For A.CED.1, use all available types of functions to create such equations, including root functions, but constrain to simple cases. While functions used in A.CED.2, 3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. Note that the example given for A.CED.4 applies to earlier instances of this standard, not to the current course.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*
• Interpret functions that arise in applications in terms of a context. *Emphasize the selection of a model function based on behavior of data and context.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Unit 3: Modeling with Functions	
Clusters and Instructional Notes	**Common Core State Standards**
• Analyze functions using different representations. *Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
• Build a function that models a relationship between two quantities. *Develop models for more complex or sophisticated situations than in previous courses.*	F.BF.1 Write a function that describes a relationship between two quantities.* b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model..*
• Build new functions from existing functions. *Use transformations of functions to find models as students consider increasingly more complex situations.* *For F.BF.3, note the effect of multiple transformations on a single graph and the common effect of each transformation across function types.* *Extend F.BF.4a to simple rational, simple radical, and simple exponential functions; connect F.BF.4a to F.LE.4.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2\,x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
• Construct and compare linear, quadratic, and exponential models and solve problems. *Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that* $\log xy = \log x + \log y$.	F.LE.4 For exponential models, express as a logarithm the solution to $a\,b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Unit 4: Inferences and Conclusions from Data

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Unit 4: Inferences and Conclusions from Data	
Clusters and Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.*	S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
• Understand and evaluate random processes underlying statistical experiments. *For S.IC.2, include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.*	S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies. *In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.* *For S.IC.4 and 5, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.*	S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.6 Evaluate reports based on data.
• Use probability to evaluate outcomes of decisions. *Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.*	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Overview of the Integrated Pathway for the Common Core State Mathematics Standards

This table shows the domains and clusters in each course in the Integrated Pathway. The standards from each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics.

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses*
Number and Quantity	The Real Number System		• Extend the properties of exponents to rational exponents. N.RN.1, 2 • Use properties of rational and irrational numbers. N.RN.3		
Number and Quantity	Quantities	• Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions* N.Q.1, 2, 3			
Number and Quantity	The Complex Number System		• Perform arithmetic operations with complex numbers. *i^2 as highest power of i* N.CN.1, 2 • Use complex numbers in polynomial identities and equations. *Quadratics with real coefficients* N.CN.7, (+) 8, (+) 9	• Use complex numbers in polynomial identities and equations. *Polynomials with real coefficients; apply N.CN.9 to higher degree polynomials* (+) N.CN. 8, 9	• Perform arithmetic operations with complex numbers. (+) N.CN.3 • Represent complex numbers and their operations on the complex plane. (+) N.CN.4, 5, 6
Number and Quantity	Vector Quantities and Matrices				• Represent and model with vector quantities. (+) N.VM.1, 2, 3 • Perform operations on vectors. (+) N.VM.4a, 4b, 4c, 5a, 5b • Perform operations on matrices and use matrices in applications. (+) N.VM.6, 7, 8, 9, 10, 11, 12

*The (+) standards in this column are those in the Common Core State Standards that are not included in any of the Integrated Pathway courses. They would be used in additional courses developed to follow Mathematics III.

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Algebra	Seeing Structure in Expressions	• Interpret the structure of expressions. *Linear expressions and exponential expressions with integer exponents* A.SSE.1a, 1b	• Interpret the structure of expressions. *Quadratic and exponential* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. *Quadratic and exponential* A.SSE.3a, 3b, 3c	• Interpret the structure of expressions. *Polynomial and rational* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. A.SSE.4	
	Arithmetic with Polynomials and Rational Expressions		• Perform arithmetic operations on polynomials. *Polynomials that simplify to quadratics* A.APR.1	• Perform arithmetic operations on polynomials. *Beyond quadratic* A.APR.1 • Understand the relationship between zeros and factors of polynomials. A.APR.2, 3 • Use polynomial identities to solve problems. A.APR.4, (+) 5 • Rewrite rational expressions. *Linear and quadratic denominators* A.APR.6, (+) 7	
	Creating Equations	• Create equations that describe numbers or relationships. *Linear, and exponential (integer inputs only); for A.CED.3, linear only* A.CED. 1, 2, 3, 4	• Create equations that describe numbers or relationships. *In A.CED.4, include formulas involving quadratic terms* A.CED. 1, 2, 4	• Create equations that describe numbers or relationships. *Equations using all available types of expressions including simple root functions* A.CED.1, 2, 3, 4	

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Algebra	Reasoning with Equations and Inequalities	• Understand solving equations as a process of reasoning and explain the reasoning. *Master linear, learn as general principle* A.REI.1 • Solve equations and inequalities in one variable. *Linear inequalities; literal that are linear in the variables being solved for;, exponential of a form, such as $2^x = 1/16$* A.REI.3 • Solve systems of equations. *Linear systems* A.REI.5, 6 • Represent and solve equations and inequalities graphically. *Linear and exponential; learn as general principle* A.REI.10, 11, 12	• Solve equations and inequalities in one variable. *Quadratics with real coefficients* A.REI.4a, 4b • Solve systems of equations. *Linear-quadratic systems* A.REI.7	• Understand solving equations as a process of reasoning and explain the reasoning. *Simple radical and rational* A.REI.2 • Represent and solve equations and inequalities graphically. *Combine polynomial, rational, radical, absolute value, and exponential functions* A.REI.11	• Solve systems of equations. (+) A.REI.8, 9
Functions	Interpreting Functions	• Understand the concept of a function and use function notation. *Learn as general principle. Focus on linear and exponential (integer domains) and on arithmetic and geometric sequences* F.IF.1, 2, 3 • Interpret functions that arise in applications in terms of a context. *Linear and exponential, (linear domain)* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear and exponential* F.IF.7a, 7e, 9	• Interpret functions that arise in applications in terms of a context. *Quadratic* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear, exponential, quadratic, absolute value, step, piecewise-defined* F.IF.7a, 7b, 8a, 8b, 9	• Interpret functions that arise in applications in terms of a context. *Include rational, square root and cube root; emphasize selection of appropriate models* F.IF.4, 5, 6 • Analyze functions using different representations. *Include rational and radical; focus on using key features to guide selection of appropriate type of model function* F.IF. 7b, 7c, 7e, 8, 9	• Analyze functions using different representations. *Logarithmic and trigonometric functions* (+) F.IF.7d

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Functions	Building Functions	• Build a function that models a relationship between two quantities. *For F.BF.1, 2, linear and exponential (integer inputs)* F.BF.1a, 1b, 2 • Build new functions from existing functions. *Linear and exponential; focus on vertical translations for exponential* F.BF.3	• Build a function that models a relationship between two quantities. *Quadratic and exponential* F.BF.1a, 1b • Build new functions from existing functions. *Quadratic, absolute value* F.BF.3, 4a	• Build a function that models a relationship between two quantities. *Include all types of functions studied* F.BF.1b • Build new functions from existing functions. *Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types* F.BF.3, 4a	• Build a function that models a relationship between two quantities. (+) F.BF.1c • Build new functions from existing functions. (+) F.BF.4b, 4c, 4d, 5
	Linear, Quadratic, and Exponential Models	• Construct and compare linear, quadratic, and exponential models and solve problems. *Linear and exponential* F.LE.1a, 1b, 1c, 2, 3 • Interpret expressions for functions in terms of the situation they model. *Linear and exponential of form* *$f(x) = b^x + k$* F.LE.5	• Construct and compare linear, quadratic, and exponential models and solve problems. *Include quadratic* F.LE. 3	• Construct and compare linear, quadratic, and exponential models and solve problems. *Logarithms as solutions for exponentials* F.LE.4	
	Trigonometric Functions		• Prove and apply trigonometric identities. F.TF.8	• Extend the domain of trigonometric functions using the unit circle. F.TF.1, 2 • Model periodic phenomena with trigonometric functions. F.TF. 5	• Extend the domain of trigonometric functions using the unit circle. (+) F.TF.3, 4 • Model periodic phenomena with trigonometric functions. (+) F.TF. 6, 7 • Prove and apply trigonometric identities. (+) F.TF. 9

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Geometry	Congruence	• Experiment with transformations in the plane. G.CO.1, 2, 3, 4, 5 • Understand congruence in terms of rigid motions. *Build on rigid motions as a familiar starting point for development of concept of geometric proof* G.CO.6, 7, 8 • Make geometric constructions. *Formalize and explain processes* G.CO.12, 13	• Prove geometric theorems. *Focus on validity of underlying reasoning while using variety of ways of writing proofs* G.CO.9, 10, 11		
	Similarity, Right Triangles, and Trigonometry		• Understand similarity in terms of similarity transformations. G.SRT.1a, 1b, 2, 3 • Prove theorems involving similarity. *Focus on validity of underlying reasoning while using variety of formats* G.SRT.4, 5 • Define trigonometric ratios and solve problems involving right triangles. G.SRT.6, 7, 8	• Apply trigonometry to general triangles. (+) G.SRT.9. 10, 11	
	Circles		• Understand and apply theorems about circles. G.C.1, 2, 3, (+) 4 • Find arc lengths and areas of sectors of circles. *Radian introduced only as unit of measure* G.C.5		

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Geometry	Expressing Geometric Properties with Equations	• Use coordinates to prove simple geometric theorems algebraically. *Include distance formula; relate to Pythagorean theorem* G.GPE. 4, 5, 7	• Translate between the geometric description and the equation for a conic section. G.GPE.1, 2 • Use coordinates to prove simple geometric theorems algebraically. *For G.GPE.4 include simple circle theorems* G.GPE.4		• Translate between the geometric description and the equation for a conic section. (+) G.GPE.3
	Geometric Measurement and Dimension		• Explain volume formulas and use them to solve problems. G.GMD.1, 3	• Visualize the relation between two-dimensional and three-dimensional objects. G.GMD.4	• Explain volume formulas and use them to solve problems. (+) G.GMD.2
	Modeling with Geometry			• Apply geometric concepts in modeling situations. G.MG.1, 2, 3	
Statistics and Probability	Interpreting Categorical and Quantitative Data	• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.1, 2, 3 • Summarize, represent, and interpret data on two categorical and quantitative variables. *Linear focus; discuss general principle* S.ID.5, 6a, 6b, 6c • Interpret linear models. S.ID.7, 8, 9		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.4	
	Making Inferences and Justifying Conclusions			• Understand and evaluate random processes underlying statistical experiments. S.IC.1, 2 • Make inferences and justify conclusions from sample surveys, experiments and observational studies. S.IC.3, 4, 5, 6	

	Domains	Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Statistics and Probability	Conditional Probability and the Rules of Probability		• Understand independence and conditional probability and use them to interpret data. *Link to data from simulations or experiments* S.CP.1, 2, 3, 4, 5 • Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.6, 7, (+) 8, (+) 9		
	Using Probability to Make Decisions		• Use probability to evaluate outcomes of decisions. *Introductory; apply counting rules* (+) S.MD.6, 7	• Use probability to evaluate outcomes of decisions. *Include more complex situations* (+) S.MD.6, 7	• Calculate expected values and use them to solve problems. (+) S.MD.1, 2, 3, 4 • Use probability to evaluate outcomes of decisions. (+) S.MD. 5a, 5b

Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics 1 uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Critical Area 4: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem in 8^{th} grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Relationships Between Quantities</td><td>• Reason quantitatively and use units to solve problems.
• Interpret the structure of expressions.
• Create equations that describe numbers or relationships.</td><td rowspan="6">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Linear and Exponential Relationships</td><td>• Represent and solve equations and inequalities graphically.
• Understand the concept of a function and use function notation.
• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.
• Interpret expressions for functions in terms of the situation they model.</td></tr>
<tr><td>Unit 3[†]
Reasoning with Equations</td><td>• Understand solving equations as a process of reasoning and explain the reasoning.
• Solve equations and inequalities in one variable.
• Solve systems of equations.</td></tr>
<tr><td>Unit 4
Descriptive Statistics</td><td>• Summarize, represent, and interpret data on a single count or measurement variable.
• Summarize, represent, and interpret data on two categorical and quantitative variables.
• Interpret linear models.</td></tr>
<tr><td>Unit 5
Congruence, Proof, and Constructions</td><td>• Experiment with transformations in the plane.
• Understand congruence in terms of rigid motions.
• Make geometric constructions.</td></tr>
<tr><td>Unit 6
Connecting Algebra and Geometry through Coordinates</td><td>• Use coordinates to prove simple geometric theorems algebraically.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

†Note that solving equations and systems of equations follows a study of functions in this course. To examine equations before functions, this unit could be merged with Unit 1.

Unit 1: Relationships Between Quantities

By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Unit 1: Relationships between Quantities	
Clusters with Instructional Notes	**Common Core State Standards**
SKILLS TO MAINTAIN *Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.*	
• Reason quantitatively and use units to solve problems. *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
• Interpret the structure of expressions. *Limit to linear expressions and to exponential expressions with integer exponents.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
• Create equations that describe numbers or relationships. *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas with a linear focus.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Represent and solve equations and inequalities graphically. *For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.*	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
• Understand the concept of a function and use function notation. *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.* *Draw examples from linear and exponential functions. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.*	F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret functions that arise in applications in terms of a context. *For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.* *N.RN.1 and N.RN. 2 will need to be referenced here before discussing exponential models with continuous domains.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100\cdot 2^n$.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
• Build a function that models a relationship between two quantities. *Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
• Build new functions from existing functions. *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.* *While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Unit 2: Linear and Exponential Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Construct and compare linear, quadratic, and exponential models and solve problems. *For F.LE.3, limit to comparisons between exponential and linear models.*	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
• Interpret expressions for functions in terms of the situation they model. *Limit exponential functions to those of the form $f(x) = b^x + k$.*	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Unit 3: Reasoning with Equations

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Unit 3: Reasoning with Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Understand solving equations as a process of reasoning and explain the reasoning. *Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.*	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
• Solve equations and inequalities in one variable. *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.*	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
• Solve systems of equations. *Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5, which requires students to prove the slope criteria for parallel lines.*	A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Unit 4: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 4: Descriptive Statistics	
Clusters with Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.*	S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
• Summarize, represent, and interpret data on two categorical and quantitative variables. *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.* *S.ID.6b should be focused on situations for which linear models are appropriate.*	S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for scatter plots that suggest a linear association.
• Interpret linear models. *Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.*	S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.9 Distinguish between correlation and causation.

Unit 5: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 5: Congruence, Proof, and Constructions	
Clusters and Instructional Notes	**Common Core State Standards**
• Experiment with transformations in the plane. *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.*	G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
• Understand congruence in terms of rigid motions. *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.*	G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
• Make geometric constructions. *Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects.* *Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.*	G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 6: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Unit 6: Connecting Algebra and Geometry Through Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Use coordinates to prove simple geometric theorems algebraically. *This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.* *Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in Mathematics I involving systems of equations having no solution or infinitely many solutions.* *G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).* G.GPE.5 Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Integrated Pathway: Mathematics II

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into 6 critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

Critical Area 2: Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Critical Area 3: Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

Critical Area 4: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Critical Area 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

Critical Area 6: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Units	Includes Standard Clusters*	Mathematical Practice Standards
Unit 1 Extending the Number System	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers. • Perform arithmetic operations with complex numbers. • Perform arithmetic operations on polynomials.	**Make sense of problems and persevere in solving them.** **Reason abstractly and quantitatively.** **Construct viable arguments and critique the reasoning of others.** **Model with mathematics.** **Use appropriate tools strategically.** **Attend to precision.** **Look for and make use of structure.** **Look for and express regularity in repeated reasoning.**
Unit 2 Quadratic Functions and Modeling	• Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems.	
Unit 3† Expressions and Equations	• Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems. • Create equations that describe numbers or relationships. • Solve equations and inequalities in one variable. • Use complex numbers in polynomial identities and equations. • Solve systems of equations.	
Unit 4 Applications of Probability	• Understand independence and conditional probability and use them to interpret data. • Use the rules of probability to compute probabilities of compound events in a uniform probability model. • Use probability to evaluate outcomes of decisions.	
Unit 5 Similarity, Right Triangle Trigonometry, and Proof	• Understand similarity in terms of similarity transformations. • Prove geometric theorems. • Prove theorems involving similarity. • Use coordinates to prove simple geometric theorems algebraically. • Define trigonometric ratios and solve problems involving right triangles. • Prove and apply trigonometric identities.	
Unit 6 Circles With and Without Coordinates	• Understand and apply theorems about circles. • Find arc lengths and areas of sectors of circles. • Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorem algebraically. • Explain volume formulas and use them to solve problems.	

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

†Note that solving equations follows a study of functions in this course. To examine equations before functions, this unit could come before Unit 2.

Unit 1: Extending the Number System

Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 2, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

Unit 1: Extending the Number System	
Clusters with Instructional Notes	**Common Core State Standards**
• Extend the properties of exponents to rational exponents.	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
• Use properties of rational and irrational numbers. *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*	N.RN.3 Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.
• Perform arithmetic operations with complex numbers. *Limit to multiplications that involve i^2 as the highest power of i.*	N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
• Perform arithmetic operations on polynomials. *Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.*	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Unit 2: Quadratic Functions and Modeling

Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Unit 2: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret functions that arise in applications in terms of a context. *Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Mathematics I on exponential functions with integer exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.* *Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.* F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
• Build a function that models a relationship between two quantities. *Focus on situations that exhibit a quadratic or exponential relationship.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Unit 2: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Build new functions from existing functions. *For F.BF.3, focus on quadratic functions and consider including absolute value functions.. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x>0$.*	F.BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4 Find inverse functions. a. Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*
• Construct and compare linear, quadratic, and exponential models and solve problems. *Compare linear and exponential growth studied in Mathematics I to quadratic growth.*	F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Unit 3: Expressions and Equations

Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

Unit 3: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret the structure of expressions. *Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
• Write expressions in equivalent forms to solve problems. *It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.*	A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
• Create equations that describe numbers or relationships. *Extend work on linear and exponential equations in Mathematics I to quadratic equations. Extend A.CED.4 to formulas involving squared variables.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*
• Solve equations and inequalities in one variable. *Extend to solving any quadratic equation with real coefficients, including those with complex solutions.*	A.REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
• Use complex numbers in polynomial identities and equations. *Limit to quadratics with real coefficients.*	N.CN.7 Solve quadratic equations with real coefficients that have complex solutions. N.CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Unit 3: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Solve systems of equations. *Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.*	A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

Unit 4: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Unit 4: Applications of Probability	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand independence and conditional probability and use them to interpret data. *Build on work with two-way tables from Mathematics I Unit 4 (S.ID.5) to develop understanding of conditional probability and independence.*	S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). S.CP.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.3 Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
• Use the rules of probability to compute probabilities of compound events in a uniform probability model.	S.CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the answer in terms of the model. S.CP.7 Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A* and *B)*, and interpret the answer in terms of the model. S.CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, *P(A* and *B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model. S.CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
• Use probability to evaluate outcomes of decisions. *This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.*	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Unit 5: Similarity, Right Triangle Trigonometry, and Proof

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem.

It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

Unit 5: Similarity, Right Triangle Trigonometry, and Proof	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand similarity in terms of similarity transformations.	G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
• Prove geometric theorems. *Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of G.CO.10 may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for G.C.3 in Unit 6.*	G.CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* G.CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* G.CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*
• Prove theorems involving similarity.	G.SRT.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
• Use coordinates to prove simple geometric theorems algebraically.	G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
• Define trigonometric ratios and solve problems involving right triangles.	G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Unit 5: Similarity, Right Triangle Trigonometry, and Proof	
Clusters and Instructional Notes	**Common Core State Standards**
• Prove and apply trigonometric identities. *In this course, limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. A course with a greater focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could continue to be limited to acute angles in Mathematics II.* *Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.*	F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

Unit 6: Circles With and Without Coordinates

In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Unit 6: Circles With an Without Coordinates	
Clusters and Instructional Notes	**Common Core State Standards**
• Understand and apply theorems about circles.	G.C.1 Prove that all circles are similar. G.C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* G.C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G.C.4 (+) Construct a tangent line from a point outside a given circle to the circle.
• Find arc lengths and areas of sectors of circles. *Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.*	G.C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
• Translate between the geometric description and the equation for a conic section. *Connect the equations of circles and parabolas to prior work with quadratic equations. The directrix should be parallel to a coordinate axis.*	G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.2 Derive the equation of a parabola given a focus and directrix.
• Use coordinates to prove simple geometric theorems algebraically. *Include simple proofs involving circles.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*
• Explain volume formulas and use them to solve problems. *Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.*	G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Integrated Pathway: Mathematics III

It is in Mathematics III that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions.[3] They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Critical Area 2: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 3: Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles open up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Critical Area 4: In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

[3]In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Inferences and Conclusions from Data</td><td>• Summarize, represent, and interpret data on single count or measurement variable.
• Understand and evaluate random processes underlying statistical experiments.
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
• Use probability to evaluate outcomes of decisions.</td><td rowspan="4">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Polynomial, Rational, and Radical Relationships.</td><td>• Use complex numbers in polynomial identities and equations.
• Interpret the structure of expressions.
• Write expressions in equivalent forms to solve problems.
• Perform arithmetic operations on polynomials.
• Understand the relationship between zeros and factors of polynomials.
• Use polynomial identities to solve problems.
• Rewrite rational expressions.
• Understand solving equations as a process of reasoning and explain the reasoning.
• Represent and solve equations and inequalities graphically.
• Analyze functions using different representations.</td></tr>
<tr><td>Unit 3
Trigonometry of General Triangles and Trigonometric Functions</td><td>• Apply trigonometry to general triangles.
• Extend the domain of trigonometric functions using the unit circle.
• Model periodic phenomena with trigonometric function.</td></tr>
<tr><td>Unit 4
Mathematical Modeling</td><td>• Create equations that describe numbers or relationships.
• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.
• Visualize relationships between two-dimensional and three-dimensional objects.
• Apply geometric concepts in modeling situations.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Inferences and Conclusions from Data

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Unit 1: Inferences and Conclusions from Data	
Clusters and Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.*	S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
• Understand and evaluate random processes underlying statistical experiments. *For S.IC.2, include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.*	S.IC.1 Understand that statistics allows inferences to be made about population parameters based on a random sample from that population. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies. *In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons., These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.* *For S.IC.4 and 5, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.*	S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.6 Evaluate reports based on data.
• Use probability to evaluate outcomes of decisions. *Extend to more complex probability models. Include situations such as those involving quality control or diagnostic tests that yields both false positive and false negative results.*	S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Unit 2: Polynomials, Rational, and Radical Relationships

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Unit 2: Polynomials, Rational, and Radical Relationships	
Clusters and Instructional Notes	**Common Core State Standards**
• Use complex numbers in polynomial identities and equations. *Build on work with quadratics equations in Mathematics II. Limit to polynomials with real coefficients.*	N.CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.* N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
• Interpret the structure of expressions. *Extend to polynomial and rational expressions.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.
• Write expressions in equivalent forms to solve problems. *Consider extending A.SSE.4 to infinite geometric series in curricular implementations of this course description.*	A.SSE.4 Derive the formula for the sum of a geometric series (when the common ratio is not 1) , and use the formula to solve problems. *For example, calculate mortgage payments.* ★
• Perform arithmetic operations on polynomials. *Extend beyond the quadratic polynomials found in Mathematics II.*	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
• Understand the relationship between zeros and factors of polynomials.	A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$. A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Unit 2: Polynomials, Rational, and Radical Relationships	
Clusters and Instructional Notes	**Common Core State Standards**
• Use polynomial identities to solve problems. *This cluster has many possibilities for optional enrichment, such as relating the example in A.APR.4 to the solution of the system $u^2+v^2=1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x+y)^{n+1} = (x+y)(x+y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.*	A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.* A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
• Rewrite rational expressions *The limitations on rational functions apply to the rational expressions in A.APR.6. A.APR.7 requires the genera division algorithm for polynomials.*	A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
• Understand solving equations as a process of reasoning and explain the reasoning. *Extend to simple rational and radical equations.*	A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
• Represent and solve equations and inequalities graphically. *Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.*	A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
• Analyze functions using different representations. *Relate F.IF.7c to the relationship between zeros of quadratic functions and their factored forms.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Unit 3: Trigonometry of General Triangles and Trigonometric Functions

Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles open up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Unit 3: Trigonometry of General Triangles and Trigonometric Functions	
Clusters and Instructional Notes	**Common Core State Standards**
• Apply trigonometry to general triangles. *With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.*	G.SRT.9 (+) Derive the formula $A = {}^{1}/_{2}\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
• Extend the domain of trigonometric functions using the unit circle.	F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
• Model periodic phenomena with trigonometric functions.	F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

Unit 4: Mathematical Modeling

In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Unit 4: Mathematical Modeling	
Clusters and Instructional Notes	**Common Core State Standards**
• Create equations that describe numbers or relationships. *For A.CED.1, use all available types of functions to create such equations, including root functions, but constrain to simple cases. While functions used in A.CED.2, 3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Mathematics I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. Note that the example given for A.CED.4 applies to earlier instances of this standard, not to the current course.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*
• Interpret functions that arise in applications in terms of a context. *Emphasize the selection of a model function based on behavior of data and context.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Unit 4: Mathematical Modeling	
Clusters and Instructional Notes	**Common Core State Standards**
• Build a function that models a relationship between two quantities. *Develop models for more complex or sophisticated situations than in previous courses.*	F.BF.1 Write a function that describes a relationship between two quantities.* b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
• Build new functions from existing functions. *Use transformations of functions to find more optimum models as students consider increasingly more complex situations.* *For F.BF.3, note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph.* *Extend F.BF.4a to simple rational, simple radical, and simple exponential functions; connect F.BF.4a to F.LE.4.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2\,x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
• Construct and compare linear, quadratic, and exponential models and solve problems. *Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x + log y.*	F.LE.4 For exponential models, express as a logarithm the solution to $a\,b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
• Visualize relationships between two-dimensional and three-dimensional objects.	G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
• Apply geometric concepts in modeling situations.	G.MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ G.MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ G.MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

High School Mathematics in Middle School[4]

There are some students who are able to move through the mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade[5] or earlier so they can take college-level mathematics in high school.[6] Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

The number of students taking high school mathematics in eighth grade has increased steadily for years. Part of this trend is the result of a concerted effort to get more students to take Calculus and other college-level mathematics courses in high school. Enrollment in both AP Statistics and AP Calculus, for example, have essentially doubled over the last decade (College Board, 2009). There is also powerful research showing that among academic factors, the strongest predictor of whether a student will earn a bachelor's degree is the highest level of mathematics taken in high school (Adelman, 1999). A recent study completed by The College Board confirms this. Using data from 65,000 students enrolled in 110 colleges, students' high school coursework was evaluated to determine which courses were closely associated with students' successful performance in college. The study confirmed the importance of a rigorous curriculum throughout a students' high school career. Among other conclusions, the study found that students who took more advanced courses, such as Pre-Calculus in the 11th grade or Calculus in 12th grade, were more successful in college. Students who took AP Calculus at any time during their high school careers were most successful (Wyatt & Wiley, 2010). And even as more students are enrolled in more demanding courses, it does not necessarily follow that there must be a corresponding decrease in engagement and success (Cooney & Bottoms, 2009, p. 2).

At the same time, there are cautionary tales of pushing underprepared students into the first course of high school mathematics in the eighth grade. The Brookings Institute's 2009 Brown Center Report on American Education found that the NAEP scores of students taking Algebra I in the eighth grade varied widely, with the bottom ten percent scoring far below grade level. And a report from the Southern Regional Education Board, which supports increasing the number of middle students taking Algebra I, found that among students in the lowest quartile on achievement tests, those enrolled in higher-level mathematics had a slightly higher failure rate than those enrolled in lower-level mathematics (Cooney & Bottoms, 2009, p. 2). In all other quartiles, students scoring similarly on achievement tests were less likely to fail if they were enrolled in more demanding courses. These two reports are reminders that, rather than skipping or rushing through content, students should have appropriate progressions of foundational content to maximize their likelihoods of success in high school mathematics.

It is also important to note that notions of what constitutes a course called "Algebra I" or "Mathematics I" vary widely. In the CCSS, students begin preparing for algebra in Kindergarten, as they start learning about the properties of operations. Furthermore, much of the content central to typical Algebra I courses—namely linear equations, inequalities, and functions—is found in the 8th grade CCSS. The Algebra I course described here ("High School Algebra I"), however, is the first formal algebra course in the Traditional Pathway (concepts from this Algebra I course are developed across the first two courses of the integrated pathway). Enrolling an eighth-grade student in a watered down version of either the Algebra I course or Mathematics I course described here may in fact do students a disservice, as mastery of algebra including attention to the Standards for Mathematical Practice is fundamental for success in further mathematics and on college entrance examinations. As mentioned above, skipping material to get students to a particular point in the curriculum will likely create gaps in the students' mathematical background, which may create additional problems later, because students may be denied the opportunity for a rigorous Algebra I or Mathematics I course and may miss important content from eighth-grade mathematics.

Middle School Acceleration

Taking the above considerations into account, as well as the recognition that there are other methods for accomplishing these goals, the Achieve Pathways Group endorses the notion that all students who are ready for rigorous high school mathematics in eighth grade should take such courses (Algebra I or Mathematics I), and that all middle schools should offer this opportunity to their students. To prepare students for high school mathematics in eighth grade, districts are encouraged to have a well-crafted sequence of **compacted courses**. The term "compacted" means to compress content, which requires a faster pace to complete, as opposed to skipping content. The Achieve Pathways Group has developed two compacted course sequences, one designed for districts using a traditional Algebra I – Geometry – Algebra II high school sequence, and the other for districts using an integrated sequence, which is commonly found internationally. Both are based on the idea that content should compact 3 years of content into 2 years, at most. In other words, compacting content from 2 years into 1 year would be too challenging, and compacting 4 years of content into 3 years starting in grade 7 runs the risk of compacting across middle and high schools. As such, grades 7, 8, and 9 were compacted into grades 7 and 8 (a 3:2 compaction). As a result, some 8th grade content is in the 7th grade courses, and high school content is in 8th grade.

[4]This section refers to mathematics content, not high school credit. The determination for high school credit is presumed to be made by state and local education agencies.
[5]Either 8th Grade Algebra I or Accelerated Mathematics I.
[6]Such as Calculus or Advanced Statistics.

The compacted traditional sequence, or, "Accelerated Traditional," compacts grades 7, 8, and High School Algebra I into two years: "Accelerated 7th Grade" and "8th Grade Algebra I." Upon successfully completion of this pathway, students will be ready for Geometry in high school. The compacted integrated sequence, or, "Accelerated Integrated," compacts grades 7, 8, and Mathematics I into two years: "Accelerated 7th Grade" and "8th Grade Mathematics I." At the end of 8th grade, these students will be ready for Mathematics II in high school. While the K-7 CCSS effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Accelerated 7th Grade course to make the 8th Grade courses more manageable.

The Achieve Pathways Group has followed a set of guidelines[7] for the development of these compacted courses.

1. **Compacted courses should include the same Common Core State Standards as the non-compacted courses.** It is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Mathematical Practices Standard.

2. **Decisions to accelerate students into the Common Core State Standards for high school mathematics before ninth grade should not be rushed.** Placing students into tracks too early should be avoided at all costs. It is not recommended to compact the standards before grade seven. In this document, compaction begins in seventh grade for both the traditional and integrated (international) sequences.

3. **Decisions to accelerate students into high school mathematics before ninth grade should be based on solid evidence of student learning.** Research has shown discrepancies in the placement of students into "advanced" classes by race/ethnicity and socioeconomic background. While such decisions to accelerate are almost always a joint decision between the school and the family, serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students.

4. **A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.** Traditionally, students taking high school mathematics in the eighth grade are expected to take Precalculus in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. Advanced courses could also include Statistics, Discrete Mathematics, or Mathematical Decision Making. An array of challenging options will keep mathematics relevant for students, and give them a new set of tools for their futures in college and career (see Fourth Courses section of this paper for further detail).

Other Ways to Accelerate Students

Just as care should be taken not to rush the decision to accelerate students, care should also be taken to provide more than one opportunity for acceleration. Some students may not have the preparation to enter a "Compacted Pathway" but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Additional opportunities for acceleration may include:

- Allowing students to take two mathematics courses simultaneously (such as Geometry and Algebra II, or Precalculus and Statistics).
- Allowing students in schools with block scheduling to take a mathematics course in both semesters of the same academic year.
- Offering summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Mathematical Practices.[8]
- Creating different compaction ratios, including four years of high school content into three years beginning in 9th grade.
- Creating a hybrid Algebra II-Precalculus course that allows students to go straight to Calculus.

A combination of these methods and our suggested compacted sequences would allow for the most mathematically-inclined students to take advanced mathematics courses during their high school career. The compacted sequences begin here:

[7] Based on work published by Washington Office of the Superintendent of Public Schools, 2008

[8] As with other methods of accelerating students, enrolling students in summer courses should be handled with care, as the pace of the courses likely be enormously fast.

Overview of the Accelerated Traditional Pathway for the Common Core State Mathematics Standards

This table shows the domains and clusters in each course in the Accelerated Traditional Pathway. The standards from each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics. For organizational purposes, clusters from 7th Grade and 8th Grade have been situated in the matrix within the high school domains.

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses*
Number and Quantity	The Real Number System	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. 7.NS.1a, 1b, 1c, 1d, 2a, 2b, 2c, 2d, 3 • Know that there are numbers that are not rational, and approximate them by rational numbers. 8.NS.1, 2 • Work with radicals and integer exponents. 8.EE.1, 2, 3, 4	• Extend the properties of exponents to rational exponents. N.RN.1, 2 • Use properties of rational and irrational numbers. N.RN.3.			
	Quantities	• Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.1, 2a, 2b, 2c, 2d, 3	• Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions* N.Q.1, 2, 3			

*The (+) standards in this column are those in the Common Core State Standards that are not included in any of the Accelerated Traditional Pathway courses. They would be used in additional courses developed to follow Algebra II.

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Number and Quantity	The Complex Number System				• Perform arithmetic operations with complex numbers. N.CN.1, 2 • Use complex numbers in polynomial identities and equations. *Polynomials with real coefficients* N.CN.7, (+) 8, (+) 9	• Perform arithmetic operations with complex numbers. (+) N.CN.3 • Represent complex numbers and their operations on the complex plane. (+) N.CN.4, 5, 6
Number and Quantity	Vector Quantities and Matrices					• Represent and model with vector quantities. (+) N.VM.1, 2, 3 • Perform operations on vectors. (+) N.VM.4a, 4b, 4c, 5a, 5b • Perform operations on matrices and use matrices in applications. (+) N.VM.6, 7, 8, 9, 10, 11, 12
Algebra	Seeing Structure in Expressions	• Use properties of operations to generate equivalent expressions. 7.EE.1, 2 • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.. 7.EE.3, 4a, 4b	• Interpret the structure of expressions. *Linear, exponential, quadratic* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. *Quadratic and exponential* A.SSE.3a, 3b, 3c		• Interpret the structure of expressions. *Polynomial and rational* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. A.SSE.4	

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Algebra	Arithmetic with Polynomials and Rational Expressions		• Perform arithmetic operations on polynomials. *Linear and quadratic* A.APR.1		• Perform arithmetic operations on polynomials. *Beyond quadratic* A.APR.1 • Understand the relationship between zeros and factors of polynomials. A.APR.2, 3 • Use polynomial identities to solve problems. A.APR.4, (+) 5 • Rewrite rational expressions. *Linear and quadratic denominators* A.APR.6, (+) 7	
	Creating Equations		• Create equations that describe numbers or relationships. *Linear, quadratic, and exponential (integer inputs only) for A.CED.3, linear only* A.CED. 1, 2, 3, 4		• Create equations that describe numbers or relationships. *Equations using all available types of expressions, including simple root functions* A.CED.1, 2, 3, 4	

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Algebra	Reasoning with Equations and Inequalities	• Understand the connections between proportional relationships, lines, and linear equations. 8.EE.5, 6 • Analyze and solve linear equations and pairs of simultaneous linear equations. 8.EE.7a, 7b	• Understand solving equations as a process of reasoning and explain the reasoning. *Master linear, learn as general principle* A.REI.1 • Solve equations and inequalities in one variable. *Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions* A.REI.3, 4a, 4b • Analyze and solve linear equations and pairs of simultaneous linear equations. 8.EE.8a, 8b, 8c • Solve systems of equations. *Linear-linear and linear-quadratic* A.REI.5, 6, 7 • Represent and solve equations and inequalities graphically. *Linear and exponential; learn as general principle* A.REI.10, 11, 12		• Understand solving equations as a process of reasoning and explain the reasoning. *Simple radical and rational* A.REI.2 • Represent and solve equations and inequalities graphically. *Combine polynomial, rational, radical, absolute value, and exponential functions* A.REI.11	• Solve systems of equations. (+) A.REI.8, 9

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Functions	Interpreting Functions		• Define, evaluate, and compare functions. 8.F.1, 2, 3 • Understand the concept of a function and use function notation. *Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences* F.IF.1, 2, 3 • Use functions to model relationships between quantities. 8.F.4, 5 • Interpret functions that arise in applications in terms of a context. *Linear, exponential, and quadratic* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear, exponential, quadratic, absolute value, step, piecewise-defined* F.IF.7a, 7b, 7e, 8a, 8b, 9		• Interpret functions that arise in applications in terms of a context. *Emphasize selection of appropriate models* F.IF.4, 5, 6 • Analyze functions using different representations. *Focus on using key features to guide selection of appropriate type of model function* F.IF.7b, 7c, 7e, 8, 9	• Analyze functions using different representations. *Logarithmic and trigonometric functions* (+) F.IF.7d

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Functions	Building Functions		• Build a function that models a relationship between two quantities. *For F.BF.1, 2, linear, exponential, and quadratic* F.BF.1a, 1b, 2 • Build new functions from existing functions. *Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only* F.BF.3, 4a		• Build a function that models a relationship between two quantities. *Include all types of functions studied* F.BF.1b • Build new functions from existing functions. *Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types* F.BF.3, 4a	• Build a function that models a relationship between two quantities. (+) F.BF.1c • Build new functions from existing functions. (+) F.BF.4b, 4c, 4d, 5
	Linear, Quadratic, and Exponential Models		• Construct and compare linear, quadratic, and exponential models and solve problems. F.LE.1a, 1b, 1c, 2, 3 • Interpret expressions for functions in terms of the situation they model. *Linear and exponential of form $f(x) = b^x + k$* F.LE.5		• Construct and compare linear, quadratic, and exponential models and solve problems. *Logarithms as solutions for exponentials* F.LE.4	
	Trigonometric Functions				• Extend the domain of trigonometric functions using the unit circle. F.TF.1, 2 • Model periodic phenomena with trigonometric functions. F.TF.5 • Prove and apply trigonometric identities. F.TF.8	• Extend the domain of trigonometric functions using the unit circle. (+) F.TF.3, 4 • Model periodic phenomena with trigonometric functions. (+) F.TF. 6, 7 • Prove and apply trigonometric identities. (+) F.TF. 9

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Geometry	Congruence	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Focus on constructing triangles* 7.G.2 • Understand congruence and similarity using physical models, transparencies, or geometric software. 8.G.1a, 1b, 1c, 2, 5 • For 8.G.5, informal arguments to establish angle sum and exterior angle theorems for triangles and angles relationships when parallel lines are cut by a transversal		• Experiment with transformations in the plane. G.CO.1, 2, 3, 4, 5 • Understand congruence in terms of rigid motions. *Build on rigid motions as a familiar starting point for development of concept of geometric proof* G.CO.6, 7, 8 • Prove geometric theorems. *Focus on validity of underlying reasoning while using variety of ways of writing proofs* G.CO.9, 10, 11 • Make geometric constructions. *Formalize and explain processes* G.CO.12,13		
	Similarity, Right Tri-angles, and Trigonom-etry	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Scale drawings* 7.G.1 • Understand congruence and similarity using physical models, transparencies, or geometric software. 8.G.3, 4, 5 • For 8.G.5, informal arguments to establish the angle-angle criterion for similar triangles		• Understand similarity in terms of similarity transformations. G.SRT.1a, 1b, 2, 3 • Prove theorems involving similarity. G.SRT.4, 5 • Define trigonometric ratios and solve problems involving right triangles. G.SRT.6, 7, 8 • Apply trigonometry to general triangles. G.SRT.9. 10, 11		

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Geometry	Circles			• Understand and apply theorems about circles. G.C.1, 2, 3, (+) 4 • Find arc lengths and areas of sectors of circles. *Radian introduced only as unit of measure* G.C.5		
	Expressing Geometric Properties with Equations			• Translate between the geometric description and the equation for a conic section. G.GPE.1, 2 • Use coordinates to prove simple geometric theorems algebraically. *Include distance formula; relate to Pythagorean theorem* G.GPE. 4, 5, 6, 7		• Translate between the geometric description and the equation for a conic section. (+) G.GPE.3
	Geometric Measurement and Dimension	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Slicing 3-D figures* 7.G.3 • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G.4, 5, 6 • Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. 8.G.9	• Understand and apply the Pythagorean theorem. *Connect to radicals, rational exponents, and irrational numbers* 8.G.6, 7, 8	• Explain volume formulas and use them to solve problems. G.GMD.1, 3 • Visualize the relation between two-dimensional and three-dimensional objects. G.GMD.4		• Explain volume formulas and use them to solve problems. (+) G.GMD.2
	Modeling with Geometry			• Apply geometric concepts in modeling situations. G.MG.1, 2, 3		

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Statistics and Probability	Interpreting Categorical and Quantitative Data		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.1, 2, 3 • Investigate patterns of association in bivariate data. 8.SP.1, 2, 3, 4 • Summarize, represent, and interpret data on two categorical and quantitative variables. *Linear focus; discuss general principle* S.ID.5, 6a, 6b, 6c • Interpret linear models. S.ID.7, 8, 9		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.4	
	Making Inferences and Justifying Conclusions	• Use random sampling to draw inferences about a population. 7.SP.1, 2 • Draw informal comparative inferences about two populations. 7.SP.3, 4			• Understand and evaluate random processes underlying statistical experiments. S.IC.1, 2 • Make inferences and justify conclusions from sample surveys, experiments and observational studies. S.IC.3, 4, 5, 6	

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Statistics and Probability	Conditional Probability and the Rules of Probability	• Investigate chance processes and develop, use, and evaluate probability models. 7.SP.5, 6, 7a, 7b, 8a, 8b, 8c		• Understand independence and conditional probability and use them to interpret data. *Link to data from simulations or experiments* S.CP.1, 2, 3, 4, 5 • Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.6, 7, (+) 8, (+) 9		
	Using Probability to Make Decisions			• Use probability to evaluate outcomes of decisions. *Introductory; apply counting rules* (+) S.MD.6, 7	• Use probability to evaluate outcomes of decisions. *Include more complex situations* (+) S.MD.6, 7	• Calculate expected values and use them to solve problems. (+) S.MD.1, 2, 3, 4 • Use probability to evaluate outcomes of decisions. (+) S.MD. 5a, 5b

Accelerated Traditional Pathway: Accelerated 7^{th} Grade

This course differs from the non-accelerated 7^{th} Grade course in that it contains content from 8^{th} grade. While coherence is retained, in that it logically builds from the 6^{th} Grade, the additional content when compared to the non-accelerated course demands a faster pace for instruction and learning. Content is organized into four critical areas, or units. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas are as follows:

Critical Area 1: Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

Critical Area 2: Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \times A$. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Critical Area 3: Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Critical Area 4: Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

<table>
<thead>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
</thead>
<tbody>
<tr><td>Unit 1
Rational Numbers and Exponents</td><td>• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
• Know that there are numbers that are not rational, and approximate them by rational numbers.
• Work with radicals and integer exponents.</td><td rowspan="4">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Proportionality and LInear Relationships</td><td>• Analyze proportional relationships and use them to solve real-world and mathematical problems.
• Use properties of operations to generate equivalent expressions.
• Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
• Understand the connections between proportional relationships, lines, and linear equations.
• Analyze and solve linear equations and pairs of simultaneous linear equations.</td></tr>
<tr><td>Unit 3
Introduction to Sampling Inference</td><td>• Use random sampling to draw inferences about a population.
• Draw informal comparative inferences about two populations.
• Investigate chance processes and develop, use, and evaluate probability models.</td></tr>
<tr><td>Unit 4
Creating, Comparing, and Analyzing Geometric Figures</td><td>• Draw, construct and describe geometrical figures and describe the relationships between them.
• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
• Understand congruence and similarity using physical models, transparencies, or geometry software.
• Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.</td></tr>
</tbody>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Rational Numbers and Exponents

Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

Unit 1: Rational Numbers and Exponents	
Clusters with Instructional Notes	**Common Core State Standards**
• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers. 7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. 7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.*

*Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Unit 1: Rational Numbers and Exponents	
Clusters with Instructional Notes	**Common Core State Standards**
• Know that there are numbers that are not rational, and approximate them by rational numbers.	8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., p2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
• Work with radicals and integer exponents.	8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$*.* 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.* 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Unit 2: Proportionality and Linear Relationships

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \times A$. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Unit 2: Proportionality and Linear Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Analyze proportional relationships and use them to solve real-world and mathematical problems.	7.RP.1 *Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks* $^1/_2$ *mile in each* $^1/_4$ *hour, compute the unit rate as the complex fraction* $^{1/2}/_{1/4}$ *miles per hour, equivalently 2 miles per hour.* 7.RP.2 *Recognize and represent proportional relationships between quantities.* a. *Decide whether two quantities are in a proportional relationship,* e.g., by testin*g for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.* b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
• Use properties of operations to generate equivalent expressions.	7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Unit 2: Proportionality and Linear Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* *7.EE.4* Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*
• Understand the connections between proportional relationships, lines, and linear equations.	8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* 8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
• Analyze and solve linear equations and pairs of simultaneous linear equations.	8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Unit 3: Introduction to Sampling and Inference

Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Unit 3: Introduction to Sampling and Inference	
Clusters with Instructional Notes	**Common Core State Standards**
• Use random sampling to draw inferences about a population.	7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. 7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
• Draw informal comparative inferences about two populations.	7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* 7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Unit 3: Introduction to Sampling and Inference	
Clusters with Instructional Notes	**Common Core State Standards**
• Investigate chance processes and develop, use, and evaluate probability models.	7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $^1/_2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* 7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Unit 4: Creating, Comparing, and Analyzing Geometric Figures

Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Unit 4: Creating, Comparing, and Analyzing Geometric Figures	
Clusters with Instructional Notes	**Common Core State Standards**
• Draw, construct, and describe geometrical figures and describe the relationships between them.	7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Unit 4: Creating, Comparing, and Analyzing Geometric Figures	
Clusters with Instructional Notes	**Common Core State Standards**
• Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. 8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
• Solve real-world and mathematical problem involving volume of cylinders, cones, and spheres.	8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

8th Grade Algebra I

The fundamental purpose of 8th Grade Algebra I is to formalize and extend the mathematics that students learned through the end of seventh grade. The critical areas, called units, deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. In addition, the units will introduce methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

This course differs from High School Algebra I in that it contains content from 8th grade. While coherence is retained, in that it logically builds from the Accelerated 7th Grade, the additional content when compared to the high school course demands a faster pace for instruction and learning.

Critical Area 1: Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Critical Area 2: Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 4: In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Critical Area 5: In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

<table>
<tr><th>Units</th><th>Includes Standard Clusters*</th><th>Mathematical Practice Standards</th></tr>
<tr><td>Unit 1
Relationships Between Quantities and Reasoning with Equations</td><td>• Reason quantitatively and use units to solve problems.
• Interpret the structure of expressions.
• Create equations that describe numbers or relationships.
• Understand solving equations as a process of reasoning and explain the reasoning.
• Solve equations and inequalities in one variable.</td><td rowspan="3">Make sense of problems and persevere in solving them.

Reason abstractly and quantitatively.

Construct viable arguments and critique the reasoning of others.

Model with mathematics.

Use appropriate tools strategically.

Attend to precision.

Look for and make use of structure.

Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 2
Linear and Exponential Relationships</td><td>• Extend the properties of exponents to rational exponents.
• Analyze and solve linear equations and pairs of simultaneous linear equations.
• Solve systems of equations.
• Represent and solve equations and inequalities graphically
• Define, evaluate, and compare functions.
• Understand the concept of a function and use function notation.
• Use functions to model relationships between quantities.
• Interpret functions that arise in applications in terms of a context.
• Analyze functions using different representations.
• Build a function that models a relationship between two quantities.
• Build new functions from existing functions.
• Construct and compare linear, quadratic, and exponential models and solve problems.
• Interpret expressions for functions in terms of the situation they model.</td></tr>
<tr><td>Unit 3
Descriptive Statistics</td><td>• Summarize, represent, and interpret data on a single count or measurement variable.
• Investigate patterns of association in bivariate data.
• Summarize, represent, and interpret data on two categorical and quantitative variables.
• Interpret linear models.</td></tr>
</table>

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Units	Includes Standard Clusters	Mathematical Practice Standards
Unit 4 Expressions and Equations	• Interpret the structure of expressions. • Write expressions in equivalent forms to solve problems. • Perform arithmetic operations on polynomials. • Create equations that describe numbers or relationships. • Solve equations and inequalities in one variable. • Solve systems of equations.	
Unit 5 Quadratics Funtions and Modeling	• Use properties of rational and irrational numbers. • Understand and apply the Pythagorean theorem. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic and exponential models and solve problems.	

Unit 1: Relationships between Quantities and Reasoning with Equations

Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions. This unit builds on earlier experiences with equations by asking students to analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Reason quantitatively and use units to solve problems. *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*[12]	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
• Interpret the structure of expressions. *Limit to linear expressions and to exponential expressions with integer exponents.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
• Create equations that describe numbers or relationships. *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variables of interest.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*
• Understand solving equations as a process of reasoning and explain the reasoning. *Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future units and courses. Students will solve exponential equations in Algebra II.*	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Unit 1: Relationships between Quantities and Reasoning with Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Solve equations and inequalities in one variable. *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.*	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 2: Linear and Exponential Functions

Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students explore systems of equations and inequalities, and they find and interpret their solutions. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	**Common Core State Standards**
• Extend the properties of exponents to rational exponents. *In implementing the standards in curriculum, these standards should occur before discussing exponential models with continuous domains.*	N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
• Analyze and solve linear equations and pairs of simultaneous linear equations. *While this content is likely subsumed by A.REI.3, 5, and 6, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.EE.8 Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
• Solve systems of equations. *Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).*	A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	**Common Core State Standards**
• Represent and solve equations and inequalities graphically. *For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.*	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
• Define, evaluate, and compare functions. *While this content is likely subsumed by F.IF.1-3 and F.IF.7a, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. 8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* 8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
• Understand the concept of a function and use function notation. *Students should experience a variety of types of situations modeled by functions Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.* *Constrain examples to linear functions and exponential functions having integral domains. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.*	F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.
• Use functions to model relationships between quantities. *While this content is likely subsumed by F.IF.4 and F.BF.1a, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret functions that arise in applications in terms of a context. *For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and exponential functions whose domain is a subset of the integers. Unit 5 in this course and Algebra II course address other types of functions.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7a, 7e, and 9 focus on linear and exponentials functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y=3^n$ and $y=100 \cdot 2^n$.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
• Build a function that models a relationship between two quantities. *Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions in F.BF.2.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
• Build new functions from existing functions. *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.* *While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	**Common Core State Standards**
• Construct and compare linear, quadratic, and exponential models and solve problems. *For F.LE.3, limit to comparisons between linear and exponential models.*	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
• Interpret expressions for functions in terms of the situation they model. *Limit exponential functions to those of the form $f(x) = b^x + k$.*	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Unit 3: Descriptive Statistics

Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 3: Descriptive Statistics	
Clusters with Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.*	S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
• Investigate patterns of association in bivariate data. *While this content is likely subsumed by S.ID.6-9, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* 8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
• Summarize, represent, and interpret data on two categorical and quantitative variables. *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.* *S.ID.6b should be focused on linear models, but may be used to preface quadratic functions in the Unit 6 of this course.*	S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.

Unit 3: Descriptive Statistics	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret linear models. *Build on students' work with linear relationship and; introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.*	S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.9 Distinguish between correlation and causation.

Unit 4: Expressions and Equations

In this unit, students build on their knowledge from unit 2, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret the structure of expressions. *Focus on quadratic and exponential expressions. For A.SSE.1b, exponents are extended from integer found in Unit 1 to rational exponents focusing on those that represent square roots and cube roots.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
• Write expressions in equivalent forms to solve problems. *Consider extending this unit to include the relationship between properties of logarithms and properties of exponents.*	A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
• Perform arithmetic operations on polynomials. *Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of* x.	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
• Create equations that describe numbers or relationships. *Extend work on linear and exponential equations in Unit 1 to include quadratic equations. Extend A.CED.4 to formulas involving squared variables.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Unit 4: Expressions and Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Solve equations and inequalities in one variable. *Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.*	A.REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
• Solve systems of equations. *Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2+y^2=1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.*	A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

Unit 5: Quadratic Functions and Modeling

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Formal work with complex numbers comes in Algebra II. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Use properties of rational and irrational numbers. *Connect N.RN.3 to physical situations, e.g., finding the perimeter of a square of area 2.*	N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
• Understand and apply the Pythagorean theorem. *Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.*	8.G.6 Explain a proof of the Pythagorean theorem and its converse. 8.G.7 Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.8 Apply the Pythagorean theorem to find the distance between two points in a coordinate system.
• Interpret functions that arise in applications in terms of a context. *Focus on quadratic functions; compare with linear and exponential functions studied in Unit 2.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Unit 5: Quadratic Functions and Modeling	
Clusters with Instructional Notes	**Common Core State Standards**
• Analyze functions using different representations. *For F.IF.7b, compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Note that this unit, and in particular in F.IF.8b, extends the work begun in Unit 2 on exponential functions with integral exponents. For F.IF.9, focus on expanding the types of functions considered to include, linear, exponential, and quadratic.* *Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.* F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
• Build a function that models a relationship between two quantities. *Focus on situations that exhibit a quadratic relationship.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
• Build new functions from existing functions. *For F.BF.3, focus on quadratic functions, and consider including absolute value functions. For F.BF.4a, focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x>0$*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* F.BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) =2\,x^3$ for $x > 0$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*
• Construct and compare linear, quadratic, and exponential models and solve problems. *Compare linear and exponential growth to growth of quadratic growth.*	F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Overview of the Accelerated Integrated Pathway for the Common Core State Mathematics Standards

This table shows the domains and clusters in each course in the Accelerated Traditional Pathway. The standards from each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics. For organizational purposes, clusters from 7th Grade and 8th Grade have been situated in the matrix within the high school domains.

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses*
Number and Quantity	The Real Number System	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. 7.NS.1a, 1b, 1c, 1d, 2a, 2b, 2c, 2d, 3 • Know that there are numbers that are not rational, and approximate them by rational numbers. 8.NS.1, 2 • Work with radicals and integer exponents. 8.EE.1, 2, 3, 4		• Extend the properties of exponents to rational exponents. N.RN.1, 2 • Use properties of rational and irrational numbers. N.RN.3.		
Number and Quantity	Quantities	• Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.1, 2a, 2b, 2c, 2d, 3	• Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions* N.Q.1, 2, 3			
Number and Quantity	The Complex Number System			• Perform arithmetic operations with complex numbers. *i^2 as highest power of i* N.CN.1, 2 • Use complex numbers in polynomial identities and equations. *Quadratics with real coefficients* N.CN.7, (+)8, (+) 9	• Use complex numbers in polynomial identities and equations. *Polynomials with real coefficients; apply N.CN.9 to higher degree polynomials* (+) N.CN.8, 9	• Perform arithmetic operations with complex numbers. (+) N.CN.3 • Represent complex numbers and their operations on the complex plane. (+) N.CN.4, 5, 6

*The (+) standards in this column are those in the Common Core State Standards that are not included in any of the Accelerated Integrated Pathway courses. They would be used in additional courses developed to follow Mathematics III.

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Number and Quantity	Vector Quantities and Matrices					• Represent and model with vector quantities. (+) N.VM.1, 2, 3 • Perform operations on vectors. (+) N.VM.4a, 4b, 4c, 5a, 5b • Perform operations on matrices and use matrices in applications. (+) N.VM.6, 7, 8, 9, 10, 11, 12
Algebra	Seeing Structure in Expressions	• Use properties of operations to generate equivalent expressions. 7.EE.1, 2 • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.. 7.EE.3, 4a, 4b	• Interpret the structure of expressions. *Linear expressions and exponential expressions with integer exponents* A.SSE.1a, 1b	• Interpret the structure of expressions. *Quadratic and exponential* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. *Quadratic and exponential* A.SSE.3a, 3b, 3c	• Interpret the structure of expressions. *Polynomial and rational* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. A.SSE.4	
	Arithmetic with Polynomials and Rational Expressions			• Perform arithmetic operations on polynomials. *Polynomials that simplify to quadratics* A.APR.1	• Perform arithmetic operations on polynomials. *Beyond quadratic* A.APR.1 • Understand the relationship between zeros and factors of polynomials. A.APR.2, 3 • Use polynomial identities to solve problems. A.APR.4, (+) 5 • Rewrite rational expressions. *Linear and quadratic denominators* A.APR.6, (+) 7	

Algebra

Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Creating Equations		• Create equations that describe numbers or relationships. *Linear and exponential (integer inputs only); for A.CED.3, linear only.* A.CED.1, 2, 3, 4	• Create equations that describe numbers or relationships. *In A.CED.4 include formulas involving quadratic terms A.CED.1, 2, 4*	• Create equations that describe numbers or relationships. *Equations using all available types of expressions, including simple root functions* A.CED.1, 2, 3, 4	
Reasoning with Equations and Inequalities	• Understand the connections between proportional relationships, lines, and linear equations. 8.EE.5, 6 • Analyze and solve linear equations and pairs of simultaneous linear equations. 8.EE.7a, 7b	• Understand solving equations as a process of reasoning and explain the reasoning. *Master linear, learn as general principle* A.REI.1 • Solve equations and inequalities in one variable. *Linear inequalities; literal equations that are linear in the variables being solved for; exponential of a form, such as $2^x = 1/16$* A.REI.3 • Analyze and solve linear equations and pairs of simultaneous linear equations *Systems of linear equations* 8.EE.8a, 8b, 8c • Solve systems of equations. *Linear systems* A.REI.5, 6 • Represent and solve equations and inequalities graphically. *Linear and exponential; learn as general principle* A.REI.10, 11, 12	• Solve equations and inequalities in one variable. *Quadratics with real coefficients* A.REI.4a, 4b • Solve systems of equations. *Linear-quadratic systems* A.REI.7	• Understand solving equations as a process of reasoning and explain the reasoning. *Simple radical and rational* A.REI.2 • Represent and solve equations and inequalities graphically. *Combine polynomial, rational, radical, absolute value, and exponential functions.* A.REI.11	• Solve systems of equations. (+) A.REI.8, 9

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Functions	Interpreting Functions		• Define, evaluate, and compare functions. 8.F.1, 2, 3 • Understand the concept of a function and use function notation. *Learn as general principle. Focus on linear and exponential (integer domains) and on arithmetic and geometric sequences* F.IF.1, 2, 3 • Use functions to model relationships between quantities. 8.F.4, 5 • Interpret functions that arise in applications in terms of a context. *Linear and exponential, (linear domain)* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear and exponential* F.IF.7a, 7e, 9	• Interpret functions that arise in applications in terms of a context. *Quadratic* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear, exponential, quadratic, absolute value, step, piecewise-defined* F.IF.7a, 7b, 8a, 8b, 9	• Interpret functions that arise in applications in terms of a context. *Include rational, square root and cube root; emphasize selection of appropriate models* F.IF.4, 5, 6 • Analyze functions using different representations. *Include rational and radical; focus on using key features to guide selection of appropriate type of model function* F.IF. 7b, 7c, 7e, 8, 9	• Analyze functions using different representations. *Logarithmic and trigonometric functions* (+) F.IF.7d

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Functions	Building Functions		• Build a function that models a relationship between two quantities. *Linear and exponential (integer inputs)* F.BF.1a, 1b, 2 • Build new functions from existing functions. *For F.BF.1, 2, linear and exponential; focus on vertical translations for exponential* F.BF.3	• Build a function that models a relationship between two quantities. *Quadratic and exponential* F.BF.1a, 1b • Build new functions from existing functions. *Quadratic, all exponential, absolute value* F.BF.3, 4a	• Build a function that models a relationship between two quantities. *Include all types of functions studied* F.BF.1b • Build new functions from existing functions. *Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types* F.BF.3, 4a	• Build a function that models a relationship between two quantities. (+) F.BF.1c • Build new functions from existing functions. (+) F.BF.4b, 4c, 4d, 5
	Linear, Quadratic, and Exponential Models		• Construct and compare linear, quadratic, and exponential models and solve problems. *Linear and exponential* F.LE.1a, 1b, 1c, 2, 3 • Interpret expressions for functions in terms of the situation they model. *Linear and exponential of form $f(x) = b^x = k$* F.LE.5	• Construct and compare linear, quadratic, and exponential models and solve problems. *Include quadratic* F.LE. 3	• Construct and compare linear, quadratic, and exponential models and solve problems. *Logarithms as solutions for exponentials* F.LE.4	
	Trigonometric Functions			• Prove and apply trigonometric identities. F.TF.8	• Extend the domain of trigonometric functions using the unit circle. F.TF.1, 2 • Model periodic phenomena with trigonometric functions. F.TF.5	• Extend the domain of trigonometric functions using the unit circle. (+) F.TF.3, 4 • Model periodic phenomena with trigonometric functions. (+) F.TF. 6, 7 Prove and apply trigonometric identities. (+) F.TF. 9

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Geometry	Congruence	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Focus on constructing triangles* 7.G.2 • Understand congruence and similarity using physical models, transparencies, or geometric software. 8.G.1a, 1b, 1c, 2, 5 • For 8.G.5, informal arguments to establish angle sum and exterior angle theorems for triangles and angles relationships when parallel lines are cut by a transversal	• Experiment with transformations in the plane. G.CO.1, 2, 3, 4, 5 • Understand congruence in terms of rigid motions. *Build on rigid motions as a familiar starting point for development of concept of geometric proof* G.CO.6, 7, 8 • Make geometric constructions. *Formalize and explain processes* G.CO.12, 13	• Prove geometric theorems. *Focus on validity of underlying reasoning while using variety of ways of writing proofs* G.CO.9, 10, 11		
	Similarity, Right Triangles, and Trigonometry	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Scale drawings* 7.G.1 • Understand congruence and similarity using physical models, transparencies, or geometric software. 8.G.3, 4, 5 • For 8.G.5, informal arguments to establish the angle-angle criterion for similar triangles		• Understand similarity in terms of similarity transformations. G.SRT.1a, 1b, 2, 3 • Prove theorems involving similarity. *Focus on validity of underlying reasoning while using variety of formats* G.SRT.4, 5 • *Define trigonometric ratios and solve problems involving right triangles.* G.SRT.6, 7, 8	• Apply trigonometry to general triangles. (+) G.SRT.9. 10, 11	

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Geometry	Circles			• Understand and apply theorems about circles. G.C.1, 2, 3, (+) 4 • Find arc lengths and areas of sectors of circles. *Radian introduced only as unit of measure* G.C.5		
	Expressing Geometric Properties with Equations		• Use coordinates to prove simple geometric theorems algebraically. *Include distance formula; relate to Pythagorean theorem* G.GPE. 4, 5, 7	• Translate between the geometric description and the equation for a conic section. G.GPE.1, 2 • Use coordinates to prove simple geometric theorems algebraically. *For G.GPE.4 include simple circle theorems* G.GPE. 4, 6		• Translate between the geometric description and the equation for a conic section. (+) G.GPE.3
	Geometric Measurement and Dimension	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Slicing 3-D figures* 7.G.3 • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G.4, 5, 6 • Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. 8.G.9	• Understand and apply the Pythagorean theorem. *Connect to radicals, rational exponents, and irrational numbers* 8.G.6, 7, 8	• Explain volume formulas and use them to solve problems. G.GMD.1, 3	• Visualize the relation between two-dimensional and three-dimensional objects. G.GMD.4	• Explain volume formulas and use them to solve problems. (+) G.GMD.2
	Modeling with Geometry				• Apply geometric concepts in modeling situations. G.MG.1, 2, 3	

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Statistics and Probability	Interpreting Categorical and Quanti-tative Data		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.1, 2, 3 • Investigate patterns of association in bivariate data. 8.SP.1, 2, 3, 4 • Summarize, represent, and interpret data on two categorical and quantitative variables. *Linear focus; discuss general principle* S.ID.5, 6a, 6b, 6c • Interpret linear models. S.ID.7, 8, 9		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.4	
	Making Infer-ences and Justifying Conclusions	• Use random sampling to draw inferences about a population. 7.SP.1, 2 • Draw informal comparative inferences about two populations. 7.SP.3, 4			• Understand and evaluate random processes underlying statistical experiments. S.IC.1, 2 • Make inferences and justify conclusions from sample surveys, experiments and observational studies. S.IC.3, 4, 5, 6	

	Domains	Accelerated 7th Grade	8th Grade Mathematics I	Mathematics II	Mathematics III	Fourth Courses
Statistics and Probability	Conditional Probability and the Rules of Probability	• Investigate chance processes and develop, use, and evaluate probability models. 7.SP.5, 6, 7a, 7b, 8a, 8b, 8c		• Understand independence and conditional probability and use them to interpret data. *Link to data from simulations or experiments* S.CP.1, 2, 3, 4, 5 • Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.6, 7, (+) 8, (+) 9		
	Using Probability to Make Decisions			• Use probability to evaluate outcomes of decisions. *Introductory; apply counting rules* (+) S.MD.6, 7	• Use probability to evaluate outcomes of decisions. *Include more complex situations* (+) S.MD.6, 7	• Calculate expected values and use them to solve problems. (+) S.MD.1, 2, 3, 4 • Use probability to evaluate outcomes of decisions. (+) S.MD. 5a, 5b

Accelerated Integrated Pathway: Accelerated 7th Grade

This course differs from the non-accelerated 7th Grade course in that it contains content from 8th grade. While coherence is retained, in that it logically builds from the 6th Grade, the additional content when compared to the non-accelerated course demands a faster pace for instruction and learning. Content is organized into four critical areas, or units. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas are as follows:

Critical Area 1: Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

Critical Area 2: Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \times A$. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Critical Area 3: Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Critical Area 4: Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Units	Includes Standard Clusters*	Mathematical Practice Standards
Unit 1 Rational Numbers and Exponents	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. • Know that there are numbers that are not rational, and approximate them by rational numbers. • Work with radicals and integer exponents.	**Make sense of problems and persevere in solving them.** **Reason abstractly and quantitatively.** **Construct viable arguments and critique the reasoning of others.** **Model with mathematics.** **Use appropriate tools strategically.** **Attend to precision.** **Look for and make use of structure.** **Look for and express regularity in repeated reasoning.**
Unit 2 Proportionality and Linear Relationships	• Analyze proportional relationships and use them to solve real-world and mathematical problems. • Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations.	
Unit 3 Introduction to Sampling and Interference	• Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use, and evaluate probability models.	
Unit 4 Creating, Comparing, and Analyzing Geometric Figures	• Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. • Understand congruence and similarity using physical models, transparencies, or geometry software. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.	

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

Unit 1: Rational Numbers and Exponents

Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. They convert between a fraction and decimal form of an irrational number. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems. They extend their mastery of the properties of operations to develop an understanding of integer exponents, and to work with numbers written in scientific notation.

<table>
<tr><th colspan="2">Unit 1: Rational Numbers and Exponents</th></tr>
<tr><th>Clusters with Instructional Notes</th><th>Common Core State Standards</th></tr>
<tr><td>• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</td><td>7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.*</td></tr>
</table>

*Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Unit 1: Rational Numbers and Exponents	
Clusters with Instructional Notes	**Common Core State Standards**
• Know that there are numbers that are not rational, and approximate them by rational numbers.	8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*
• Work with radicals and integer exponents.	8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Unit 2: Proportionality and Linear Relationships

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \times A$. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation.

Unit 2: Proportionality and Linear Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Analyze proportional relationships and use them to solve real-world and mathematical problems.	7.RP.1 *Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour.* 7.RP.2 *Recognize and represent proportional relationships between quantities.* a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. 7.RP.3 *Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
• Use properties of operations to generate equivalent expressions.	7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. 7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Unit 2: Proportionality and Linear Relationships	
Clusters with Instructional Notes	**Common Core State Standards**
• Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* *7.EE.4* Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*
• Understand the connections between proportional relationships, lines, and linear equations.	8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* 8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
• Analyze and solve linear equations and pairs of simultaneous linear equations.	8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Unit 3: Introduction to Sampling and Inference

Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Unit 3: Introduction to Sampling and Inference	
Clusters with Instructional Notes	**Common Core State Standards**
• Use random sampling to draw inferences about a population.	7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. 7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
• Draw informal comparative inferences about two populations.	7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* 7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Unit 3: Introduction to Sampling and Inference	
Clusters with Instructional Notes	**Common Core State Standards**
• Investigate chance processes and develop, use, and evaluate probability models.	7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $^1/_2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* 7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Unit 4: Creating, Comparing, and Analyzing Geometric Figures

Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity, they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Unit 4: Creating, Comparing, and Analyzing Geometric Figures	
Clusters with Instructional Notes	**Common Core State Standards**
• Draw, construct, and describe geometrical figures and describe the relationships between them.	7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
• Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Unit 4: Creating, Comparing, and Analyzing Geometric Figures	
Clusters with Instructional Notes	**Common Core State Standards**
• Understand congruence and similarity using physical models, transparencies, or geometry software.	8.G.1 Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. 8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
• Solve real-world and mathematical problem involving volume of cylinders, cones, and spheres.	8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

8th Grade Mathematics I

The fundamental purpose of 8th Grade Mathematics I is to formalize and extend the mathematics that students learned through the end of seventh grade. Content in this course is grouped into six critical areas, or units. The units of study deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend. 8th Grade Mathematics 1 includes an exploration of the role of rigid motions in congruence and similarity. The Pythagorean theorem is introduced, and students examine volume relationships of cones, cylinders, and spheres. 8th Grade Mathematics 1 uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

This course differs from Mathematics I in that it contains content from 8th grade. While coherence is retained, in that it logically builds from Accelerated 7th Grade, the additional content when compared to the high school course demands a faster pace for instruction and learning.

Critical Area 1: Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions.

Critical Area 2: Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds on earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions.

Critical Area 4: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Units	Includes Standard Clusters*	Mathematical Practice Standards
Unit 1 Relationships Between Quantities	• Reason quantitatively and use units to solve problems. • Interpret the structure of expressions. • Create equations that describe numbers or relationships.	**Make sense of problems and persevere in solving them.** **Reason abstractly and quantitatively.** **Construct viable arguments and critique the reasoning of others.** **Model with mathematics.** **Use appropriate tools strategically.** **Attend to precision.** **Look for and make use of structure.** **Look for and express regularity in repeated reasoning.**
Unit 2 Linear and Exponential Relationships	• Represent and solve equations and inequalities graphically. • Define, evaluate, and compare functions. • Understand the concept of a function and use function notation. • Use functions to model relationships between quantities. • Interpret functions that arise in applications in terms of a context. • Analyze functions using different representations. • Build a function that models a relationship between two quantities. • Build new functions from existing functions. • Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model.	
Unit 3[†] Reasoning with Equations	• Understand solving equations as a process of reasoning and explain the reasoning. • Solve equations and inequalities in one variable. • Analyze and solve linear equations and pairs of simultaneous linear equations. • Solve systems of equations.	
Unit 4 Descriptive Statistics	• Summarize, represent, and interpret data on a single count or measurement variable. • Investigate patterns of associate in bivariate data. • Summarize, represent, and interpret data on two categorical and quantitative variables. • Interpret linear models.	
Unit 5 Congruence, Proof, and Constructions	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. • Make geometric constructions. • Understand and apply the Pythagorean theorem.	
Unit 6 Connecting Algebra and Geometry through Coordinates	• Use coordinates to prove simple geometric theorems algebraically.	

*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

[†]Note that solving equations and systems of equations follows a study of functions in this course. To examine equations before functions, this unit could be merged with Unit 1.

Unit 1: Relationships Between Quantities

Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations, and functions.

Unit 1: Relationships between Quantities	
Clusters with Instructional Notes	**Common Core State Standards**
• Reason quantitatively and use units to solve problems. *Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.*	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
• Interpret the structure of expressions. *Limit to linear expressions and to exponential expressions with integer exponents.*	A.SSE.1 Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
• Create equations that describe numbers or relationships. *Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas which are linear in the variables of interest.*	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Unit 2: Linear and Exponential Functions

Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	**Common Core State Standards**
• Represent and solve equations and inequalities graphically. *For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.*	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
• Define, evaluate, and compare functions. *While this content is likely subsumed by F.IF.1-3 and F.IF.7a, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. 8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* 8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*
• Understand the concept of a function and use function notation. *Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.* *Constrain examples to linear functions and exponential functions having integral domains. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.*	F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	**Common Core State Standards**
• Use functions to model relationships between quantities. *While this content is likely subsumed by F.IF.4 and F.BF.1a, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
• Interpret functions that arise in applications in terms of a context. *For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and exponential functions whose domain is a subset of the integers. Mathematics II and III will address other types of functions.* *N.RN.1 and N.RN. 2 will need to be referenced here before discussing exponential functions with continuous domains.*	F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
• Analyze functions using different representations. *For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \times 2^n$.*	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
• Build a function that models a relationship between two quantities. *Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and connect geometric sequences to exponential functions in F.BF.2.*	F.BF.1 Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Unit 2: Linear and Exponential Functions	
Clusters with Instructional Notes	**Common Core State Standards**
• Build new functions from existing functions. *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.* *While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.*	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
• Construct and compare linear, quadratic, and exponential models and solve problems. *For F.LE.3, limit to comparisons to those between exponential and linear models.*	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
• Interpret expressions for functions in terms of the situation they model. *Limit exponential, with exponential functions to those of the form $f(x) = b^x + k$.*	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Unit 3: Reasoning with Equations

This unit builds on earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions.

Unit 3: Reasoning with Equations	
Clusters with Instructional Notes	**Common Core State Standards**
• Understand solving equations as a process of reasoning and explain the reasoning. *Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations in Mathematics III.*	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
• Solve equations and inequalities in one variable. *Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.*	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
• Analyze and solve linear equations and pairs of simultaneous linear equations. *While this content is likely subsumed by A.REI.3, 5, and 6, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.EE.8 Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
• Solve systems of equations. *Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).*	A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Unit 4: Descriptive Statistics

Students use regression techniques to describe relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 4: Descriptive Statistics	
Clusters with Instructional Notes	**Common Core State Standards**
• Summarize, represent, and interpret data on a single count or measurement variable. *In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.*	S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
• Investigate patterns of association in bivariate data. *While this content is likely subsumed by S.ID.6-9, it could be used for scaffolding instruction to the more sophisticated content found there.*	8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* 8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
• Summarize, represent, and interpret data on two categorical and quantitative variables. *Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.* *S.ID.6b should be focused on situations for which linear models are appropriate.*	S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.

Unit 4: Descriptive Statistics	
Clusters with Instructional Notes	**Common Core State Standards**
• Interpret linear models. *Build on students' work with linear relationship and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure how well the data fit the relationship. The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9.*	S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.9 Distinguish between correlation and causation.

Unit 5: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 5: Congruence, Proof, and Constructions	
Clusters with Instructional Notes	**Common Core State Standards**
• Experiment with transformations in the plane. *Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.*	G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
• Understand congruence in terms of rigid motions. *Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.*	G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
• Make geometric constructions. *Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects.* *Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.*	G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
• Understand and apply the Pythagorean theorem. *Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.*	8.G.6 Explain a proof of the Pythagorean theorem and its converse. 8.G.7 Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.8 Apply the Pythagorean theorem to find the distance between two points in a coordinate system.

Unit 6: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean Theorem to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Unit 6: Connecting Algebra and Geometry Through Coordinates	
Clusters with Instructional Notes	**Common Core State Standards**
• Use coordinates to prove simple geometric theorems algebraically. *Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.* *Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in Mathematics I involving systems of equations having no solution or infinitely many solutions.* *G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.*	G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2). G.GPE.5 Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Additional Mathematics Courses

The “college and career ready” line has been based on evidence from a number of sources, including international benchmarking, surveys of postsecondary faculty and employers, review of state standards, and expert opinion. Students meeting these standards should be well-prepared for introductory mathematics courses in 2- and 4- year colleges. Still, there are persuasive reasons for students to continue on to take a fourth mathematics course in high school.

Research consistently finds that taking mathematics above the Algebra II level highly corresponds to many measures of student success. In his groundbreaking report Answers in the Toolbox, Clifford Adelman found that the strongest predictor of postsecondary success is the highest level of mathematics completed (Executive Summary). ACT has found that taking more mathematics courses correlates with greater success on their college entrance examination. Of students taking (Algebra I, Geometry and Algebra II and no other mathematics courses), only thirteen percent of those students met the benchmark for readiness for college algebra. One additional mathematics course greatly increased the likelihood that a student would reach that benchmark, and three-fourths of students taking Calculus met the benchmark (ACTb 13).

Students going through the pathways should be encouraged to select from a range of high quality mathematics options. STEM-intending students should be strongly encouraged to take Precalculus and Calculus (and perhaps a computer science course). A student interested in psychology may benefit greatly from a course in discrete mathematics, followed by AP Statistics. A student interested in starting a business after high school could use knowledge and skills gleaned from a course on mathematical decision-making. Mathematically-inclined students can, at this level, double up on courses—a student taking college calculus and college statistics would be well-prepared for almost any postsecondary career.

Taken together, there is compelling rationale for urging students to continue their mathematical education throughout high school, allowing students several rich options once they have demonstrated mastery of core content. The Pathways describe possible courses for the first three years of high school. Other arrangements of the Common Core State Standards for high school are possible. Standards marked with a (+) may appear either in courses required for all students, or in later courses. In particular, the (+) standards can form the starting point for fourth year courses in Precalculus and in Probability and Statistics. Other fourth year courses, for example Calculus, Modeling, or Discrete Mathematics are possible.

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